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ANALYSIS OF MATHEMATICAL ABILITIES REQUIRED FOR SUCCESS IN NINTH-GRADE MATHEMATICS.

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CERTAIN TYPES OF INTELLECTUAL ABILITIES WHICH DEFINE THE MATHEMATICAL APTITUDES OF NINTH-GRADE STUDENTS AND WHICH RELATE TO ACHIEVEMENT IN SPECIFIC MATHEMATICS COURSES WERE DEFINED AND EVALUATED. ANALYSES OF VARIANCE AND COVARIANCE INDICATED THE RANGE OF VARIATION IN EIGHT TEST SCORE DISTRIBUTIONS OF VERBAL AND MATHEMATICAL APTITUDES AMONG FIVE SEPARATE GROUPS OF NINTH-GRADE STUDENTS. FACTOR ANALYSES SUPPORTED THE MAIN HYPOTHESIS THAT MATHEMATICAL ABILITY IS COMPRISED OF A NUMBER OF APTITUDES AND NOT SIMPLY A UNITARY TRAIT. FINDINGS SHOWED THAT THE NATURE OF MATHEMATICAL APTITUDES AS SPECIFIC AND INDEPENDENT TRAITS OR AS CORRELATED TRAITS WAS STILL UNDETERMINED AND WOULD REQUIRE FURTHER RESEARCH. (GD)

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Research Project Number:
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The Research reported herein was performed pursuant to a contract with the Office of Education, U. S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

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Preface

A preliminary note about the events which prompted this study may provide some perspective on its purposes. In reading this report one might presume that it was authored by a mathematics educator, and prompted by a general interest in mathematics curricula. Contrary to this, the motivation for this study originated in an interest in the nature of learning abilities, and was precipitated by experiences with individual students.

The questions which arose pertained to the application of curriculum policies to specific students as follows: Should a student who has received a C or lower in a 9th grade algebra course be admitted to a course in Geometry? Should a student in the lowest quintile of a mathematics placement test be programmed in a 9th grade mathematics course; should he receive any further course work in high school mathematics? The first policy existed in a particular school and may prevail in many schools. The other policies were under consideration in another system. These policies had evolved from teachers' judgments about the value of mathematics courses for certain students, and the degree of success which they might be expected to attain. Data are generally not available, however, to support or refute their judgments.

These programming policies evoked additional questions as follows: Should Geometry be considered sequential to algebra; or should it be regarded as an independent area of study? Is mathematical aptitude a general aptitude; or, are there specific abilities, some of which might enable a student to achieve satisfactorily in geometry despite unsatisfactory achievement in algebra? Are the mathematical abilities of students entering the 9th grade sufficiently mature and stable to permit reliable prediction of their future mathematics achievements? Obviously definitive answers to these questions could not be obtained in a single study. Also, inasmuch as they refer to mathematics curricula which span over several years, and they refer to abilities which evolve during the course of mental development, longitudinal studies would appear to be necessary.

There were several factors, however, which contraindicated longitudinal research. First, since the introduction of the so-called modern mathematics courses in the curricula the topic contents of mathematics courses have been considerably changed. The sequence of courses, e.g., arithmetic, introductory algebra, plane geometry, solid geometry, trigonometry and advanced algebra, which existed in the traditional mathematics curriculum, has generally been reorganized. Modern mathematics has emphasized the study of related concepts and the discovery of mathematics principles, rather than the mastery of separate content areas. As changes in the mathematics curriculum proceed, a consideration of the sequential nature of the courses, and how these relate to students' abilities will eventually become necessary. Second, since the mathematics curriculum is still in the process of being changed, and since several tracks of modern and traditional mathematics may be offered at a given grade level, it seemed preferable to conduct a cross sectional study of the types of abilities which students present at a particular grade level.
The next logical step then is to examine how these abilities are related to their performance in the specific courses in which they are enrolled.

Preliminary discussion about these problems was carried on with Dr. Nancy Whitman, Associate Professor in Mathematics Education at the University of Hawaii and with Miss Amy Tsunehiro, graduate student and intern teacher. The plans for this study were then formulated. Preliminary endorsement to conduct this study was obtained from Dr. William Savard, Assistant Superintendent of Research, and from Miss Beatrice Loui, Coordinator of Testing Services of the State Department of Education. Administrative approval and assistance were then obtained from Mr. Donald Leach, principal of the Stevenson Intermediate School.

Miss Sharon Kim, Research Assistant in the Education Research and Development Center, was employed to coordinate the study. She established an excellent liaison between the cooperating school and the Education Research Center, and her services on the project were invaluable.

Particular gratitude is expressed to Mrs. Jennie Nakamoto, Chairman of the Mathematics Department and to Mr. Maurice Edwards, Vice Principal at Stevenson School for planning and arranging the students' schedules and for their assistance in facilitating the data collection. The study, as it was eventually carried out, required extensive assistance and cooperation from the mathematics teachers and from the ninth grade students at Stevenson School. Their assistance is also acknowledged gratefully.

Donald A. Leton
23 November 1966

Chapter 1

Introduction

Mathematics, which originated in the common needs of daily life, has now expanded into many abstract branches. Although the abstract features of mathematics have given rise to a philosophical idea about its purity, nevertheless these theories can only be validated by regularities in the phenomena of the natural world and the universe. Unfortunately as the higher levels of abstraction have been developed in mathematics they also became more removed from common knowledge.

Because of its abstractness mathematics is said to be more difficult to teach than any other subject field. On the other hand it has also assumed more and more importance in modern day affairs. Our technical and scientific culture has again asserted the need for mathematics knowledge. Along with this demand for mathematics knowledge there has also occurred a desire for good mathematics teaching. Mathematics educators are now faced with three significant tasks: 1. the selection of mathematics content from kindergarten through graduate programs, 2. the arrangement of curricular sequences, and 3. the evaluation of mathematics content in general education curricula and in specific vocational and professional curricula.

Purposes

There were two general purposes for this study. The first was to identify the types of intellectual abilities which define the mathematical aptitudes of ninth grade students; and the second was to determine how these abilities are related to achievement in specific mathematics courses.

The point of view that a variety of intellectual abilities evolve during the course of mental development is now generally accepted. The nature and number of such abilities, however, is the subject of much theoretical and research debate (e.g., Guilford 1959, French 1963, Thurstone 1933, Thurstone 1941 and others). There are contrasting views as to the nature of numerical intelligence. For example Wechsler (1944) regards it as a type of verbal intelligence. In the Wechsler Intelligence Scales arithmetical reasoning ability is assessed in the Arithmetic Subtest of the Verbal Scales. In contrast to this, Thurstone (1941), in her study of the primary mental abilities, identified a numerical factor independent of verbal comprehension and word fluency. Although numerical ability is commonly recognized as an independent trait the question of whether it differentiates with various other subabilities, and when such differentiation occurs, has generally not been investigated.

It has been presumed that the ability to perceive space-and-form relationships, and logical reasoning ability are components of geometry aptitude. Spatial and form-perception abilities appear to evolve rather early in mental development (e.g., Thurstone 1933, Frostig et al 1964, Leton 1963); however, their influence on subsequent arithmetic achievement, or their possible influence on achievement in modern mathematics courses at the elementary school level has not as yet been investigated.

Inasmuch as algebra achievement requires the manipulation of abstract symbols it may be hypothesized that abstract reasoning ability would be more predictive of success in algebra than would abilities such as numerical intelligence and space-form perception.

Grouping of students on the basis of prior grades introduces a complex set of predictor variables. Inasmuch as academic instruction is largely a verbal transaction, intellectual traits such as verbal comprehension and general reasoning have been found to be highly predictive of grades. Students are also rewarded, however, for displaying certain personal and behavioral characteristics, e.g., conformity, responsibility, and intellectual efficiency (Walters 1962). Since a major proportion of the variance in course grades is due to these personal and behavioral traits, and since they are not assessed in the usual academic aptitude tests, course grades do not represent a suitable criterion of the efficacy of ability predictors.

On the one hand scholastic aptitude is viewed as a general trait, and for school-age children is considered to be synonymous with intelligence. When tests are used for decisions about school admission and curriculum placement, most frequently they are tests of general scholastic aptitude. On the other hand the existence of specific and group factors of intelligence have been demonstrated in research. The school curricula are differentiated into tracks and levels. Relatively little attention has been given to the use of external criteria such as course achievements to validate specific intelligence factors. Also, the extent to which differences in course achievements such as algebra and geometry depend upon the differences in the patterns of mental abilities presented by students is still unknown.

Chapter 2

Review of Research

A variety of new courses, e.g., UICSM, SMSG, are being introduced into the mathematics curriculum which have not been evaluated on the basis of comparisons with traditional courses, nor on the basis of the abilities required for their mastery. Perhaps the greatest problem in the organizations of present as well as revised curricula is that these are not examined in the light of the students' aptitudes for various kinds of mathematics achievement.

Prediction Studies

A number of studies have attempted to predict success for a specific course at a particular grade level. In predicting success in first year algebra various studies have used either algebra grades or algebra achievement tests as criterion variables. Dinkel's study (1959) is most pertinent among recent research pertaining to the prediction of algebra grades. In the first year of his study he found a multiple correlation of .78 between previous grades, intelligence, mathematics aptitude and achievement tests with grades in algebra. Using the same variables the second year he obtained a multiple correlation of .86. In a more recent study Callicut (1961) used a combination of intelligence and achievement scores with subject grades from eighth grade courses to predict grades in ninth grade algebra. He found that arithmetic grades from the eighth grade and the arithmetic achievement scores from the Stanford Achievement Battery were the best predictors.

Final grades for eighth grade mathematics were found by Barnes and Asher (1962) to be the best single predictor of final grades in algebra (r = .58). By combining the eighth grade final grades with the scores on the Iowa Every-Pupil Test of Basic Skills and the Orleans Algebra Prognosis Test to predict achievement, they obtained a multiple correlation of .63. Combining three additional predictor variables increased the multiple correlation to .66.

Using a different approach, Cain (1966) compared the relationships of the Verbal Reasoning and Numerical Ability subtests of the Differential Aptitude Test Battery to algebra achievement. His sample was partitioned according to science curricular groups, modern biology and traditional biology. In both groups the correlations of the Numerical Ability subtest with the algebra grades were significantly higher than the Verbal Reasoning subtest.

In other studies of algebra achievement using grades as the criterion, multiple correlations ranged from .57 (Clifton, 1940) to .86 (Layton, 1941). Torgerson (1933) found a multiple correlation of .60 and Ayers (1934) obtained a multiple correlation of .70.

Using algebra achievement tests as criterion measures, Osburn and Melton (1963) compared the predictive validities of various measures for traditional algebra classes and experimental (modern) algebra classes. In both courses there were no significant differences in the proficiency of prediction by the aptitude tests. The coefficients of correlations between aptitude and achievement tests for both groups ranged from .62 to .66. "Spatial and mechanical reasoning tests were more valid for the experimental course than for the traditional course...the DAT spelling tests gave characteristically higher validities in the traditional course."

Duncan (1961) studied performance in eighth grade algebra using intelligence, algebra aptitude test scores, arithmetic grade placement scores and interest scores from science and literature scales as predictors. Performance on the Seattle Algebra Test was the criterion. He obtained a multiple correlation coefficient of .76. In other studies predicting performance on a first year algebra achievement test multiple correlation coefficients ranged from .65 to .84 (Dickter, 1933; Grover, 1932; Guiler, 1944; and McCuen, 1930).

Several other researchers studied algebra achievement as well as achievement in other mathematics courses. Lee and Hughes (1934) studied prediction of algebra and geometry achievement. They found that for both algebra and geometry, the aptitude test predicted scores on the respective achievement tests better than they predicted final course grades. Teacher ratings of student ability predicted final grades better than final achievement test scores. The predictions of achievement scores from aptitude tests were more successful than the predictions of final grades from the teachers' ratings.

Tempero and Ivanoff (1960) studied ninth grade general mathematics and beginning algebra. They correlated end-of-year achievement tests with SCAT Verbal, Quantitative, and Total scores. For both curricular groups the SCAT Quantitative correlated higher with the criterion than did the Verbal subtest, but the Verbal subtest had a higher predictive validity for the year end achievement in the algebra group than for the achievement in the general mathematics group. In one of the early studies on this problem Seagoe (1938) compared the predictive validities of intelligence, aptitude and achievement tests for general mathematics and algebra. She found that for both curricular groups the Stanford Arithmetic Achievement Test was superior to both the aptitude tests and the general intelligence tests.

Analyses of Mathematical Aptitudes

Even though a number of researchers have found differences in algebra, arithmetic and geometry aptitudes (Lee, 1955; Oldham, 1938; and Holzinger and Swineford, 1946) most of the predictive studies assume a general factor of mathematical ability. Oldham's studies (1937, 1938) illustrate the problems involved in analyzing mathematical aptitudes. In her factor analyses of mathematical abilities she identified a group factor between arithmetic and geometry which stems from the application of number concepts to geometry problems. She also concluded that a group factor between arithmetic and algebra is due to the ability to compute accurately, and to perceive geometric forms. The use of geometric perception in the analysis of algebra aptitudes raises the question as to the amount and the sequence of earlier instruction in mathematics.

A mathematical group factor was also found by Wrigley (1958). He concludes that "...the different branches of mathematics are linked together more closely by a mathematical group factor than they would be if a general factor only were in operation." In addition, he isolated a general intelligence factor, verbal and spatial factors and a numerical group factor.

"Performance in geometry is connected with spatial ability as measured by the spatial factor. Performance in arithmetic (especially mechanical arithmetic) and to a lesser extent performance in algebra, is in part dependent upon numerical ability as measured by a number factor."

In her factor analysis of mathematics ability Sister M. Canisia's (1962) oblique solution produced 12 factors. In a second-order analysis she obtained 4 factors.

> The findings of this study suggest that mathematical thought processes appear to be mainly processes of education, organization, and manipulation of relations. Mathematical thinking seems to be characterized by a fluency and flexibility of thought material under restricting conditions such as are often imposed by the assumptions, postulates, and definitions of a mathematical problem. The number factor appears to be quite unrelated to the other factors in terms of which mathematical ability was described in this study. (Sister M. Canisia, 1962: 125)

Kline (1956) administered a battery of 38 tests to each of two successive intermediate algebra classes and factor analyzed the two sets of data separately. The factors congruent to both analyses were then determined. There were 5 factors considered to be congruent in the two studies: Verbal Comprehension, Deductive Reasoning, Algebraic Manipulative Skill, Number Ability, and Adaptability to a New Task.

> Factor A, Verbal Comprehension, was obviously present in those algebra tests which required the conversion of verbal statements into algebraic symbols and in the test of informational ability of algebra.
>
> Factor B, Deductive Reasoning, was noticeably present in tests of the following intermediate algebra topics: exponents, binomial theorem, progressions, use of tables, Cartesian graphs...

> Factor C, Algebraic Manipulative Skill,...was definitely in evidence in tests of the following intermediate algebra topics: fractions, factoring, quadratic equations, radicals, exponents, and simultaneous equations.
>
> Factor D, Number Ability, had no loadings above .30 in any of the algebra tests. If the study had included more tests of practical problems, as in logarithms, progressions, and numerical trigonometry, instead of emphasizing the theory, there is a good possibility that the number factor would have been stronger in the algebra tests.
>
> Factor E, not present in the algebra tests, is more difficult to identify.... The author has labeled it Adaptability to New Tasks, because all the tests with significant loadings on the factor required the student to perform a task he had never done before. (Kline, 1956: 69-71)

Guilford, Hoepfner, and Peterson (1965), in their study of ninth grade mathematics, compared the predictive validities of standard academic aptitude tests, i.e., CTMM, Iowa and DAT, with the predictive validities of measures of structure-of-intellect factors. The ninth grade mathematics courses under study included Basic Mathematics, Non-College Algebra, Regular Algebra, and Accelerated Algebra.

Thirteen factors were obtained from a principal-axes analysis of 28 test variables. The factor scores were slightly better for the prediction of achievement in regular algebra, whereas the 9 standard tests were better for predicting achievement in basic mathematics. When the 13 structure-of-intellect factors were combined with the 9 standard predictors, significantly higher multiple correlations occurred for both the accelerated and regular algebra course, but not for the basic mathematics course.

Discriminant analysis of the 25 tests used in the structure-of-intellect factor study differentiated the successful algebra from the successfu mathematics students. The tests which discriminated between the above-median success groups were not entirely the same as those which were selected in stepwise prediction as best predictors of the entire ranges of achievement in algebra and mathematics classes. Guilford (1965) concludes that at least 10 factors are necessary for a predictive battery (for the ninth grade mathematics curriculum studied) and 3 additional tests are necessary to cover the task of classification. He concludes that 12 different factors, primarily from the symbolic category of the structure-of-intellect and dealing with the products of relationships and implications, were most relevant for ninth grade mathematics. Some of these factors are represented in the earlier studies by Oldham (1937, 1938), Kline (1956), Wrigley (1958), and Canisia (1962). The question, however, of how many general and how many group factors may reside in mathematical aptitude does not appear to be adequately resolved as yet.

Guilford raises an additional question as to whether achievement in the "new" mathematics courses will take the same kind of predictor variables as have been found for the traditional types of courses. Although some consideration of this question appears in the study by Osburn and Melton (1963) the factor structure of mathematical prognosis tests, and the relationship of such factors to success in modern mathematics curricula have not been determined as yet.

Chapter 3

Procedures

Population and Sample

The subjects for this study were the ninth grade students from Robert Louis Stevenson Intermediate School in Honolulu, Hawaii. This school was selected as representative of intermediate schools in Hawaii in terms of its mathematics curricula, and in terms of the characteristics of its students. The students are from a variety of community backgrounds, and they present a wide range of ability and achievement levels.

There were 227 boys and 249 girls entering as ninth grade students in the Fall of 1965. Because of student transfers there was an attrition of 16 boys and 22 girls from this study. All of the ninth grade students were programmed in mathematics courses, 33 were in the University of Illinois Committee on School Mathematics (UICSM) class, 120 were in Algebra 1 & 2 classes, 90 in Mathematics 9X, 120 in Mathematics 9A and 75 in Mathematics 9U. A description of the courses is included in the appendix.

Instruments

The predictor tests administered to the students included the Orleans Algebra Prognosis Test (OAPT), the Orleans Geometry Prognosis Test (OGPT), the Cooperative Mathematics Test (CMT) and the Differential Aptitude Test Battery (DAT). Other predictor variables included scores from the Quantitative subtest of the School and College Ability Tests, Form 3A (SCAT), the total scores of the Mathematics subtests of the Sequential Tests of Educational

Progress, Form 3A (STEP), and grades from the eighth grade mathematics course. End-of-course grades and teacher-assigned stanines were used as criterion variables for the total group. Criterion tests for the specific mathematics courses were the UICSM final exam for the UICSM class, the Cooperative Algebra Test for the algebra classes, and the arithmetic test of the SRA Achievement Battery for the 9X, 9A, and 9U classes.

The SCAT, STEP, DAT, CMT and Cooperative Algebra Test were administered by school personnel as part of their test program. The teachers in each of the curriculum tracks assigned students within each course to stanine groups.

Analyses

Means and standard deviations of the distributions of students' intelligence and achievement test scores were computed for each of the mathematics curricula. Analysis of variance was used to determine the extent of variation in verbal and numerical abilities among the groups. Analysis of covariance was utilized to determine the extent of variation in mathematical aptitudes which prevailed after adjusting for the initial variation in verbal intelligence.

Product-moment correlations were computed for the aptitude and achievement test variables. The variables included Numerical Aptitude, Abstract Reasoning and Space Relations test of the DAT battery, the Quantitative subtest of the SCAT, the Mathematics test of the STEP, and the subtests of the Cooperative Mathematics, the Orleans Algebra Prognosis, and the Orleans Geometry Prognosis. The resulting correlation matrices were factor analyzed.

Stepwise regression analyses were computed to select subsets of the variables to predict the following criteria: 1) teacher-assigned stanines, 2) final grades, and 3) performances on the end-of-course achievement tests.

Chapter 4

Results

Analyses

Summary statistics for the distributions of aptitude test scores and for grades in eighth grade mathematics were computed. The results for each curricula track, for boys and girls, and for the total ninth grade class are shown in tables 1 through 12. Graphs of the means and standard deviations of the mathematics tests for the five curriculum groups are included with the tables.

Analyses of Variance in Mathematics Aptitudes

Students were assigned to the mathematics curricula on the basis of several criteria. These included an ability criterion, the SCAT Total score; an achievement criterion, previous grade in eighth grade mathematics; and a prognosis criterion, the CMT. Subsequent modifications of these assignments were also made on the basis of teacher recommendations and students' and parents' requests. These criteria were not used sequentially, nor objectively; so the overlap which eventually appeared between groups on these distributions was high.

Inasmuch as the initial placements in the five curricular groups were not systematic,the differences between the groups in terms of their general verbal and quantitative abilities were unknown. Analysis of variance and covariance procedures were employed to study the extent of the variations

Table 1

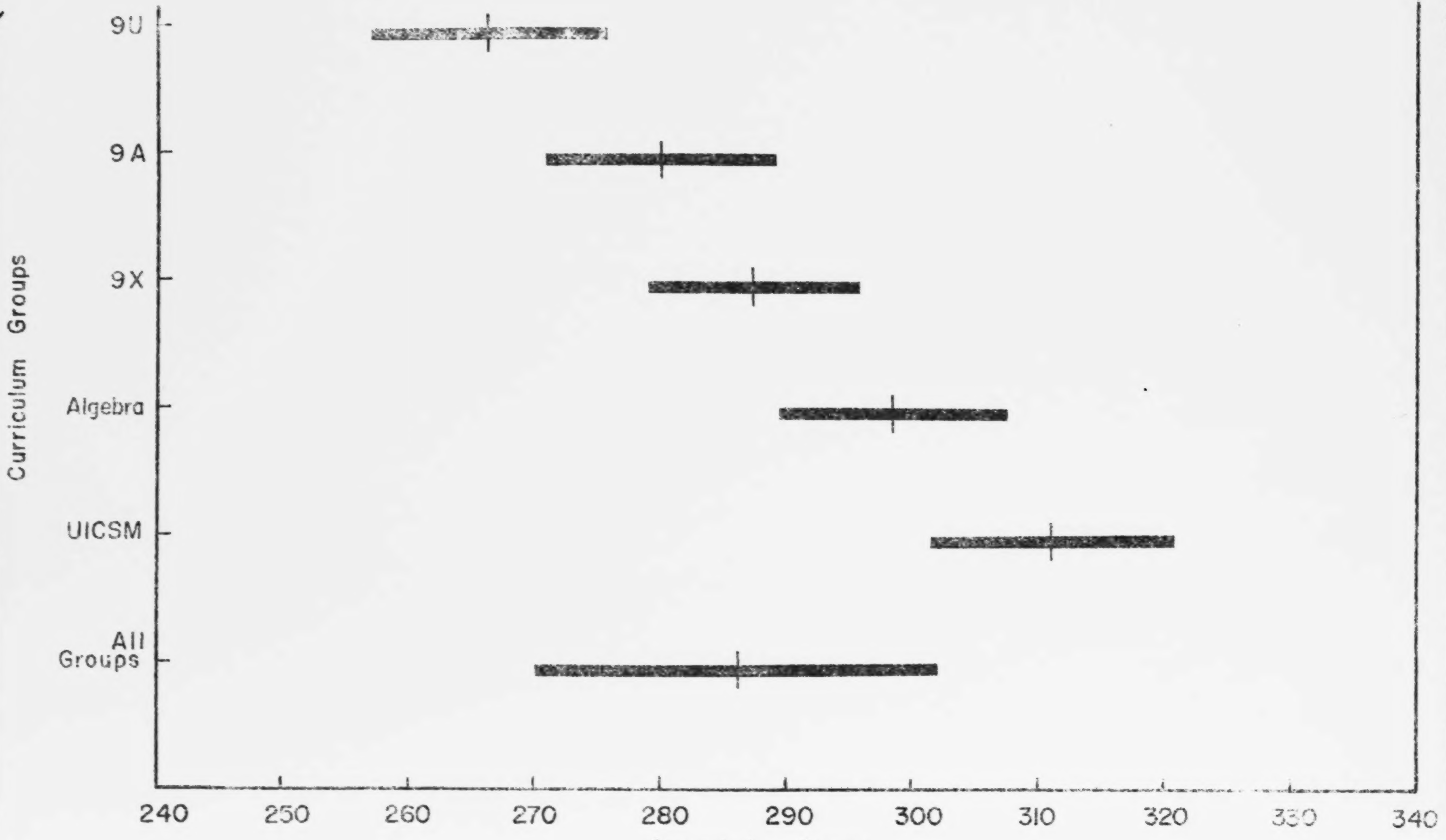
Summary Statistics for Distribution of SCAT and STEP Scores

Test and Subtest	UICSM			Algebra			9X			9A			9U		
	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.
SCAT															
Verbal	34	295.2	20.50	124	278.2	10.86	92	271.5	8.35	126	263.5	9.57	91	254.0	8.20
Quantitative	34	311.1	9.67	124	298.7	9.24	92	287.6	8.40	126	280.2	9.18	91	266.7	9.41
Total	34	297.7	6.63	124	286.8	7.21	92	279.3	5.55	126	272.4	6.74	91	262.6	6.55
STEP															
Reading	34	298.7	7.51	128	289.9	10.38	92	283.8	9.85	126	273.6	12.70	89	255.3	13.39
Writing	34	295.4	13.47	127	284.1	11.32	92	277.0	10.50	125	267.8	12.11	91	253.6	12.31
Mathematics	34	288.9	6.77	125	277.4	7.56	92	267.7	6.19	126	261.3	8.18	91	249.3	10.35

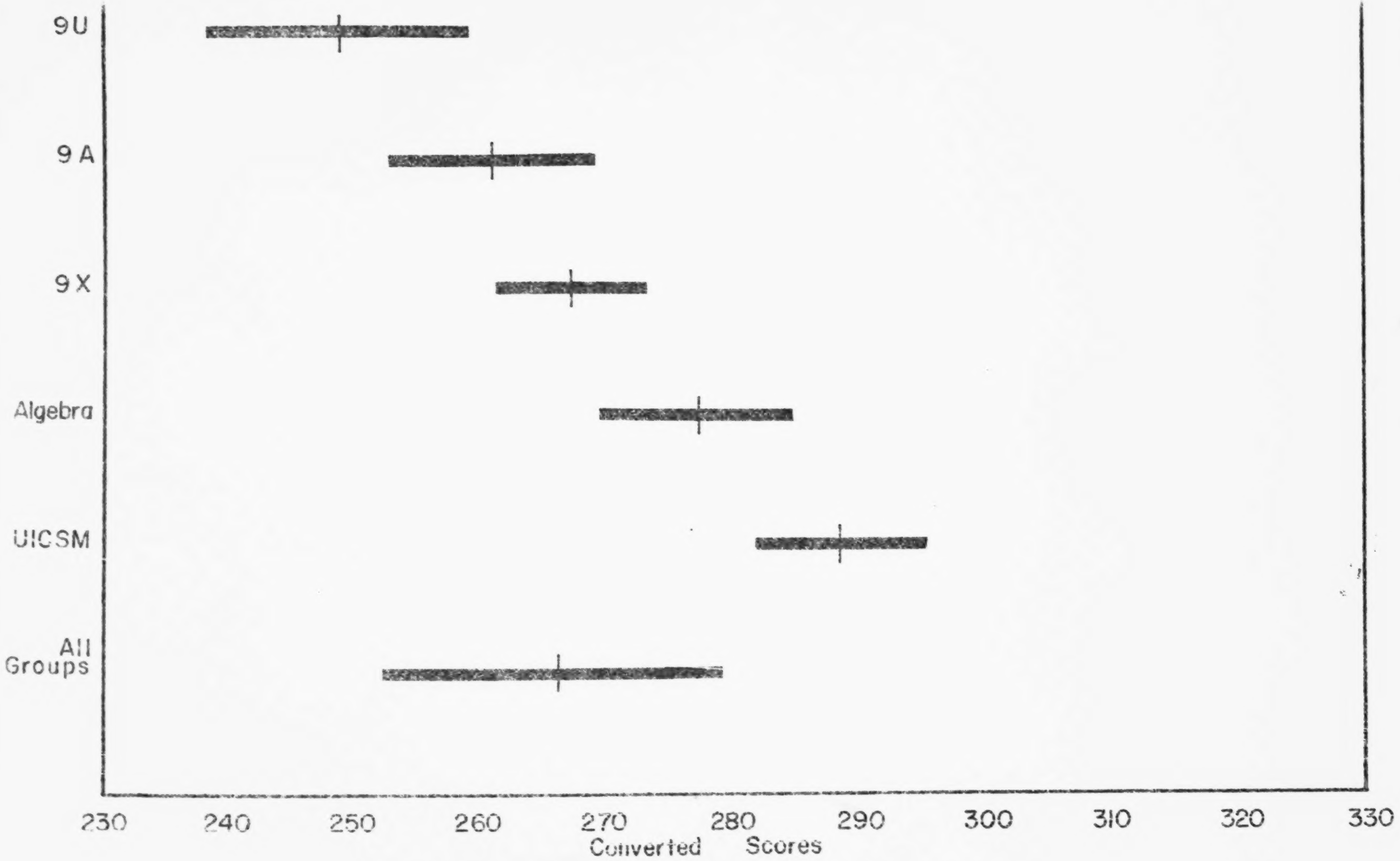
Table 2

Summary Statistics for Distribution of SCAT and STEP Scores

Test and Subtest	Boys			Girls			Total		
	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.	N	Converted Score Mean	S.D.
SCAT									
Verbal	225	269.2	15.76	242	269.7	15.12	467	269.4	15.42
Quantitative	225	286.0	16.81	242	286.4	15.10	467	286.2	15.93
Total	225	277.4	12.82	242	277.7	11.49	467	277.5	12.14
STEP									
Reading	227	275.7	18.62	242	280.9	16.14	469	278.4	17.57
Writing	227	270.9	17.38	242	275.5	16.44	469	273.3	17.04
Mathematics	226	268.1	13.97	242	265.0	14.04	468	266.5	14.07



GRAPH OF MEANS AND S.D.'S OF SCAT QUANTITATIVE
SUBTEST FOR FIVE CURRICULA GROUPS



GRAPH OF MEANS AND S.D.'S OF STEP MATH SUBTEST FOR FIVE CURRICULA GROUPS

Table 3

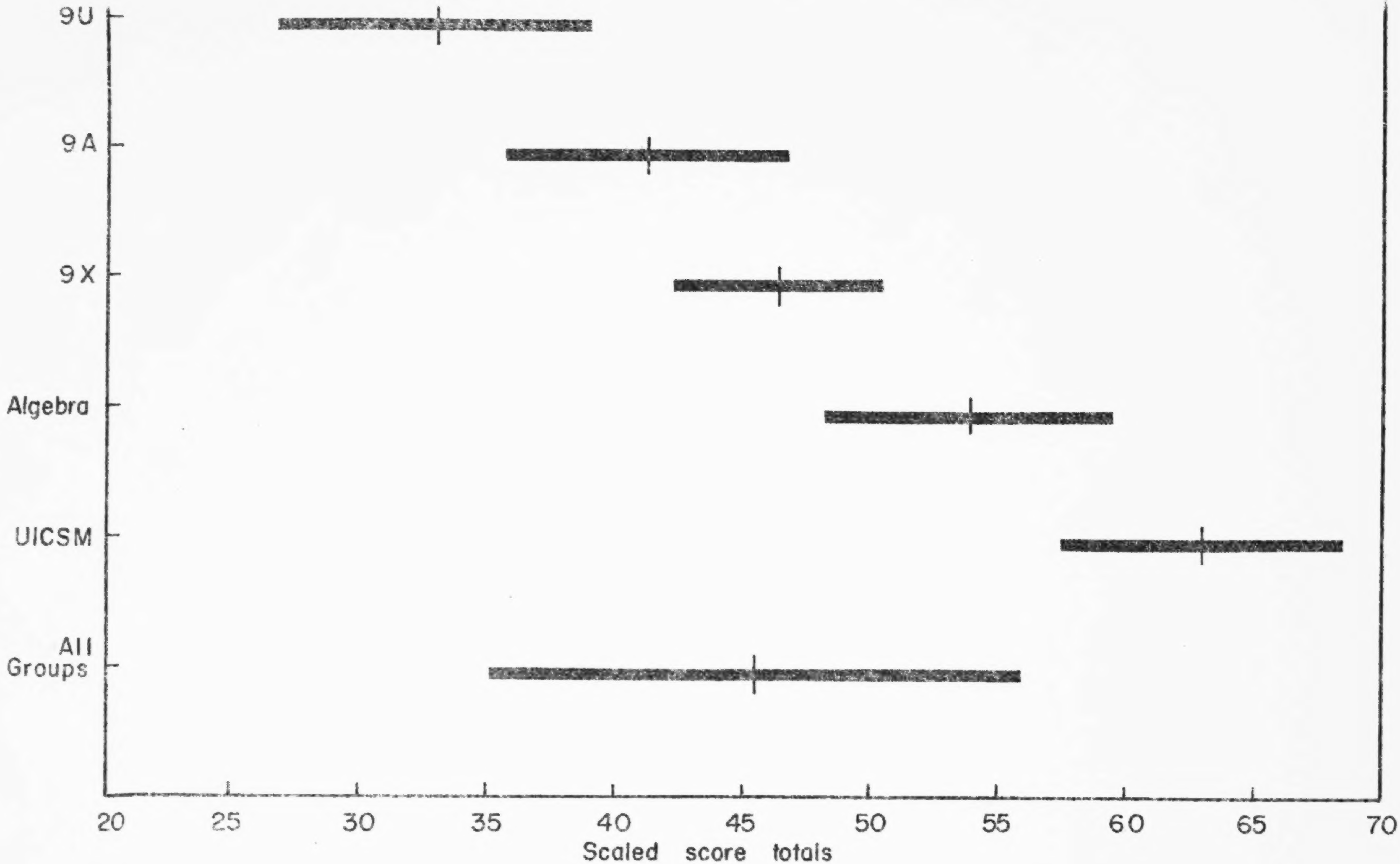
Summary Statistics for Distributions of Cooperative Mathematics Test Scores

Test	UICSM			Alg			9X			9A			9U		
	N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.
I. Skills	34	62.2	7.76	129	52.3	6.19	92	45.1	4.58	125	42.9	5.50	93	34.5	6.13
II. Facts & Concepts	34	60.1	9.36	129	52.7	6.38	92	47.5	5.36	125	41.8	6.50	93	33.7	6.53
III. Application	34	61.9	5.02	128	52.8	8.43	92	43.4	8.14	125	35.9	11.30	93	29.4	10.73
IV. Appreciation	34	66.9	9.33	128	57.6	8.08	92	48.8	7.15	125	44.1	8.57	93	34.3	10.15
Total	34	63.0	5.59	128	53.9	5.57	92	46.4	4.09	125	41.3	5.49	93	33.0	6.14

Table 4

Summary Statistics for Distributions of Cooperative Mathematics Test Scores

Test		Boys			Girls			Total		
		N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.	N	Scaled Score Mean	S.D.
I.	Skills	227	46.0	10.18	247	45.3	9.04	474	45.6	9.60
II.	Facts & Concepts	227	46.3	11.01	247	45.0	9.27	474	45.6	10.15
III.	Applications	227	44.5	14.31	246	40.6	13.21	473	42.5	13.87
IV.	Appreciation	227	48.8	13.72	246	48.0	11.97	473	48.4	12.83
	Total	227	46.5	11.09	246	44.8	9.48	473	45.6	10.31



GRAPH OF MEANS AND S.D.'S OF COOPERATIVE
MATHEMATICS TEST FOR FIVE CURRICULA GROUPS

Table 5

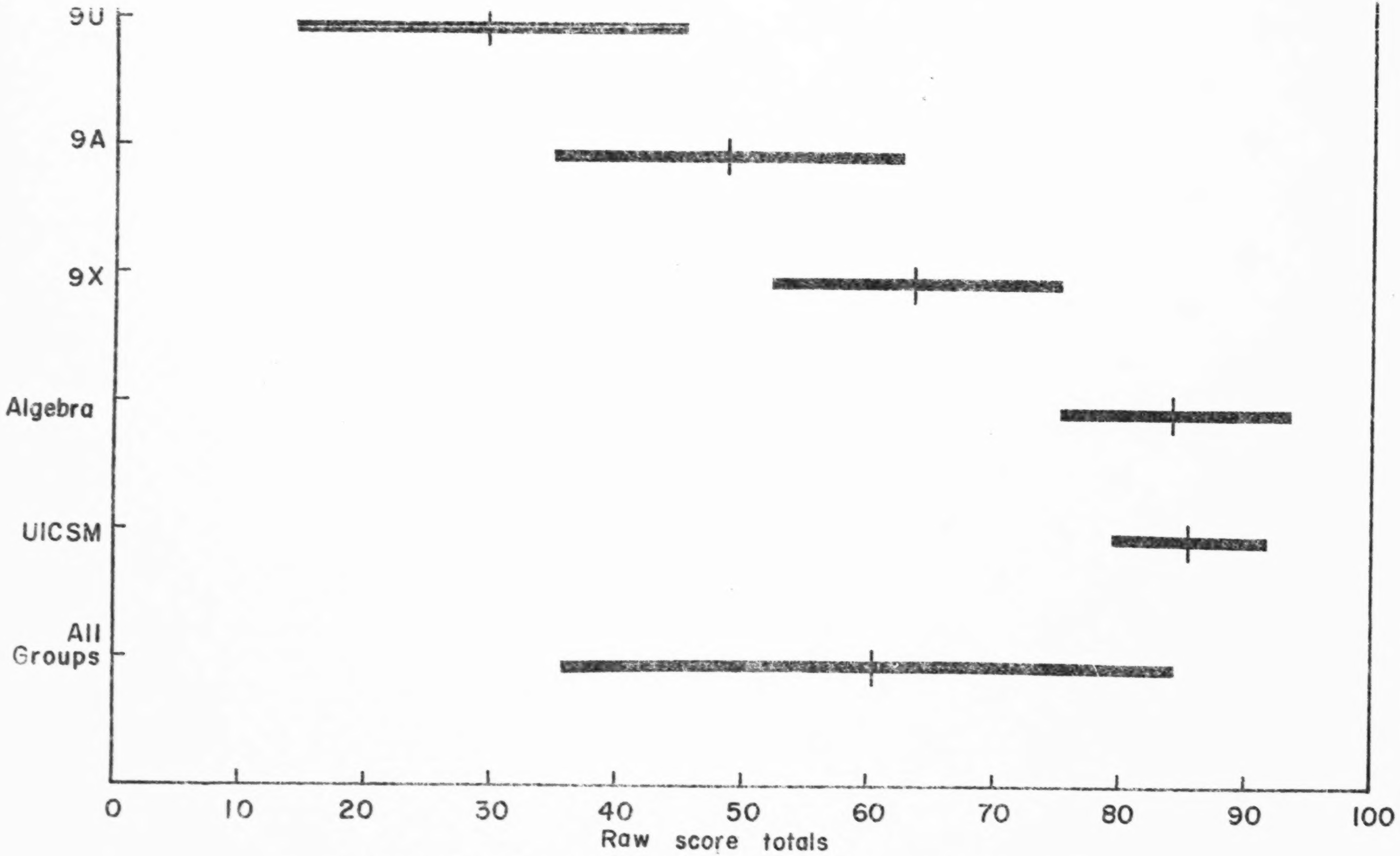
Summary Statistics for Orleans Algebra Prognosis Test

Subtest	UICSM			Algebra			9X			9A			9U		
	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Arithmetic Test	33	6.818	1.87	132	6.113	1.99	91	4.780	2.12	129	3.697	2.13	87	1.896	1.75
Test 1	33	7.969	0.17	132	7.840	0.42	91	7.230	1.36	129	6.565	1.69	87	4.977	2.71
Test 2	33	5.969	0.17	132	5.977	0.14	91	5.747	0.73	129	5.627	0.92	87	4.023	2.13
Test 3	33	7.818	0.39	132	7.469	1.05	91	7.065	1.14	129	5.589	2.21	87	3.092	2.23
Test 4	33	5.575	0.56	132	5.462	0.81	91	4.670	1.43	129	4.217	1.60	87	2.103	2.18
Test 5	33	5.757	0.50	132	5.477	0.92	91	4.516	1.57	129	2.845	1.86	87	1.712	1.75
Test 6	33	6.757	0.50	132	6.181	1.06	91	4.780	1.95	129	3.108	2.35	87	1.839	2.35
Test 7	33	7.909	0.52	132	7.598	0.85	91	6.362	1.88	129	4.689	2.16	87	3.137	1.98
Test 8	33	12.424	3.39	132	12.022	3.82	91	6.241	4.23	129	3.395	3.66	87	1.195	1.68
Test 9	33	12.121	3.63	132	13.409	2.88	91	7.912	3.61	129	5.503	3.01	87	3.678	2.62
Test 10	33	6.575	0.96	132	6.863	1.20	91	4.758	1.21	129	3.682	1.63	87	2.011	1.65
Total	33	85.697	6.14	132	84.378	9.23	91	63.923	11.47	129	48.930	14.04	87	29.632	15.53

Table 6

Summary Statistics for the Orleans Algebra Prognosis Test

Subtests	Total			Boys			Girls		
	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Arithmetic Test	472	4.468	2.56	224	4.325	2.64	248	4.596	2.48
Test 1	472	6.855	1.89	224	6.817	1.95	248	6.891	1.84
Test 2	472	5.476	1.29	224	5.388	1.37	248	5.556	1.21
Test 3	472	6.095	2.33	224	6.080	2.32	248	6.108	2.35
Test 4	472	4.358	1.89	224	4.205	1.98	248	4.496	1.79
Test 5	472	3.898	2.10	224	3.991	2.05	248	3.814	2.14
Test 6	472	4.311	2.56	224	4.303	2.75	248	4.318	2.38
Test 7	472	5.764	2.41	224	5.973	2.38	248	5.576	2.42
Test 8	472	6.582	5.59	224	6.482	5.82	248	6.673	5.39
Test 9	472	8.305	4.88	224	8.383	5.05	248	8.233	4.73
Test 10	472	4.673	2.26	224	4.705	2.40	248	4.645	2.14
Total	472	60.747	23.83	224	60.656	25.09	248	60.830	22.698



GRAPH OF MEANS AND S.D.'S FOR ORLEANS ALGEBRA PROGNOSIS TEST FOR FIVE CURRICULA GROUPS

Table 7

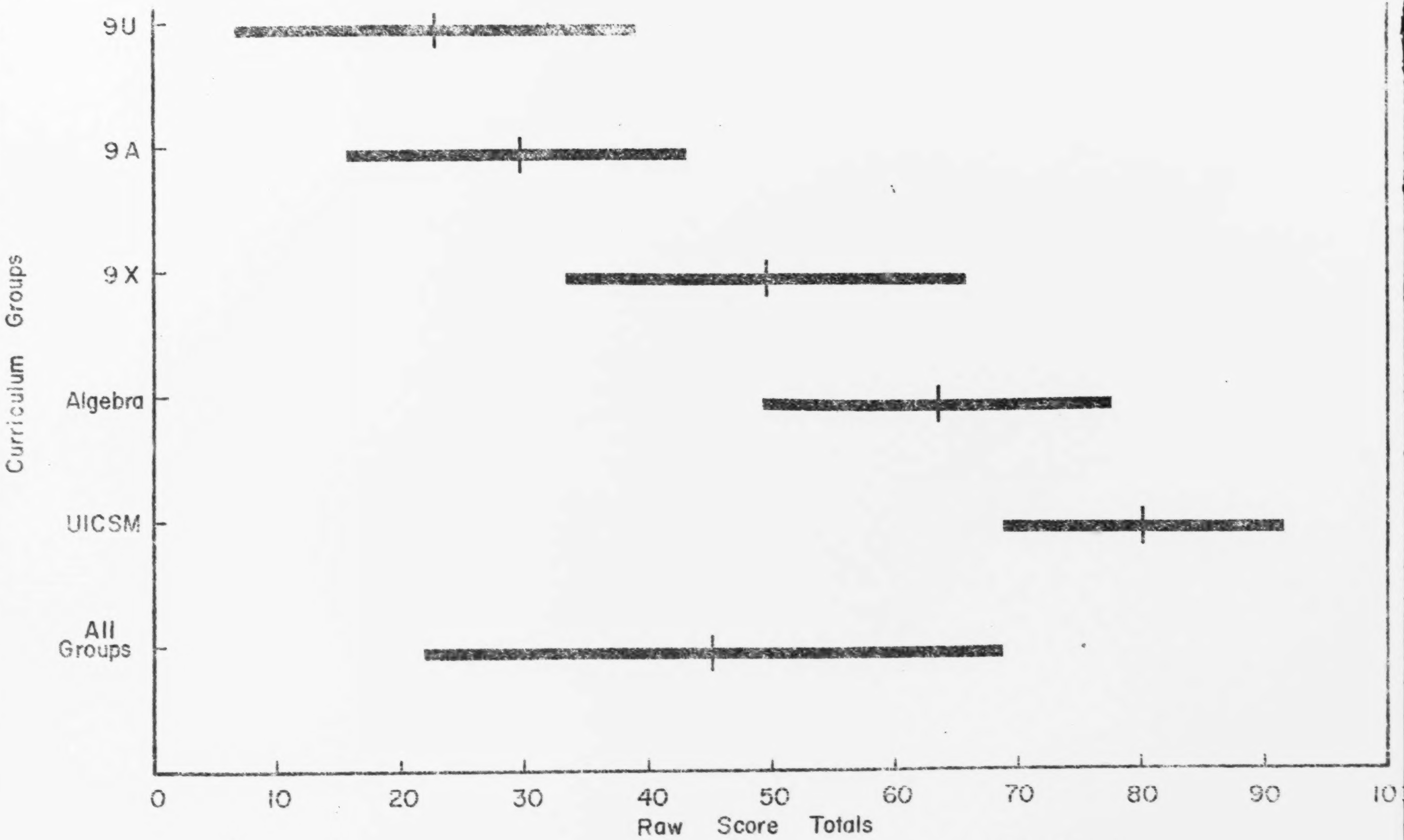
Summary Statistics for Orleans Geometry Prognosis Test

Subtest	UICSM			Algebra			9X			9A			9U		
	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Test 1	33	6.545	1.27	132	4.984	1.87	91	3.967	1.97	129	2.000	1.98	87	1.597	2.29
Test 2	33	7.272	1.25	132	5.689	2.09	91	4.560	2.39	129	2.596	2.45	87	1.620	2.09
Test 3	33	11.090	2.76	132	11.174	1.24	91	9.802	2.89	129	7.186	4.12	87	4.448	4.13
Test 4	33	10.393	.96	132	9.742	1.78	91	8.033	2.61	129	5.503	2.40	87	3.908	2.98
Test 5	33	9.272	1.28	132	7.568	5.96	91	5.560	6.46	129	2.674	2.09	87	2.712	2.31
Test 6	33	5.121	1.36	132	3.348	1.85	91	2.142	1.85	129	0.658	1.02	87	0.551	1.18
Test 7	33	5.121	1.79	132	4.295	1.85	91	3.637	1.62	129	1.945	1.51	87	1.770	1.64
Test 8	33	15.697	4.90	132	9.409	5.47	91	6.813	4.98	129	3.891	3.71	87	4.114	4.14
Test 9	33	9.515	1.71	132	7.583	2.30	91	5.538	2.79	129	2.945	2.44	87	2.160	2.47
Total	33	80.030	11.41	132	63.242	14.18	91	49.461	16.42	129	29.410	13.78	87	22.781	16.24

Table 8

Summary Statistics for Orleans Geometry Prognosis Test

Subtest	Total			Boys			Girls		
	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Test 1	472	3.457	2.54	224	3.656	2.72	248	3.278	2.35
Test 2	472	3.987	2.83	224	4.075	2.77	248	3.907	2.89
Test 3	472	8.574	4.07	224	9.000	3.95	248	8.189	4.15
Test 4	472	7.224	3.30	224	7.571	3.24	248	6.911	3.32
Test 5	472	5.067	5.07	224	5.116	3.24	248	5.024	6.29
Test 6	472	1.989	2.08	224	2.285	2.19	248	1.721	1.95
Test 7	472	3.118	2.05	224	3.303	2.17	248	2.951	1.93
Test 8	472	6.864	5.72	224	7.366	6.24	248	6.411	5.17
Test 9	472	5.057	3.45	224	5.415	3.49	248	4.733	3.38
Total	472	45.055	23.60	224	47.781	24.41	248	42.592	22.61



GRAPH OF MEANS AND S.D.'S OF ORLEANS GEOMETRY PROGNOSIS TEST FOR FIVE CURRICULA GROUPS

Table 9

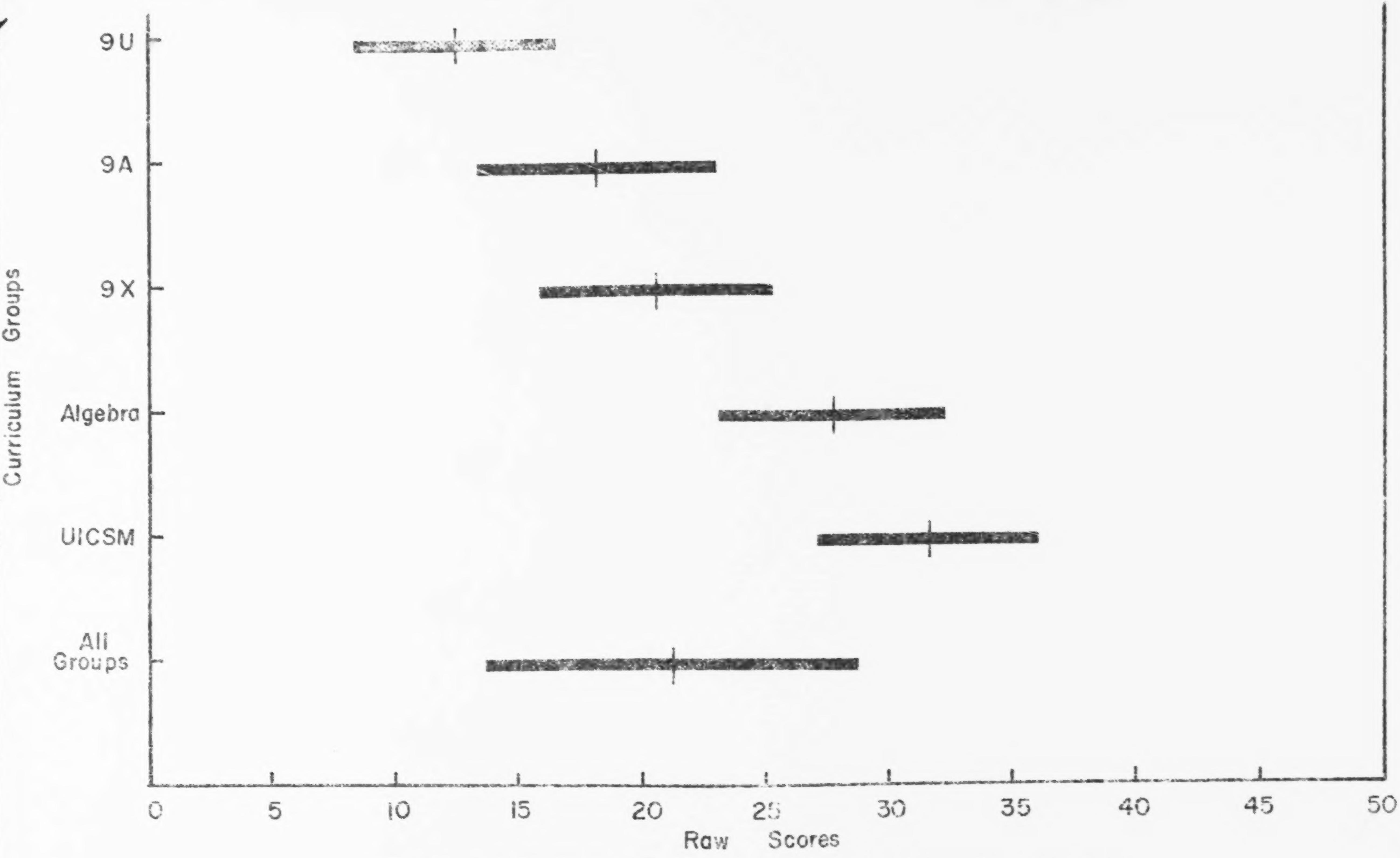
Summary Statistics for the Differential Aptitude Test

	UICSM			Algebra			9X			9A			9U		
Subtest	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Verbal Reasoning	34	40.647	4.63	131	32.183	7.67	93	25.182	7.25	127	18.748	7.01	88	11.920	4.72
Numerical Ability	34	31.617	4.59	131	27.778	4.69	93	20.612	4.78	127	18.275	4.90	89	12.516	4.11
Abstract Reasoning	34	41.470	7.34	131	37.801	7.43	93	34.236	8.33	127	29.881	10.23	88	21.465	9.52
Space Relations	34	46.000	8.57	132	37.704	10.70	93	31.161	10.68	127	26.763	9.82	89	20.258	7.51
Mechanical Reasoning	34	50.411	6.89	132	44.621	8.16	93	40.516	7.42	127	38.007	7.81	88	34.000	7.86
Clerical Speed & Accuracy	34	54.029	10.81	131	52.129	8.98	93	49.978	11.88	127	48.228	11.70	89	41.483	12.99
Language Usage Spelling	34	87.000	11.18	132	83.424	10.88	93	74.462	13.71	127	66.299	13.20	89	53.865	11.91
Language Usage Grammar	34	42.911	6.42	132	35.931	8.16	93	30.118	7.23	127	24.874	7.16	89	16.719	6.15
Verbal Reas. + Numerical Ability	34	72.323	5.93	131	59.961	9.85	93	45.591	10.21	127	37.023	10.09	88	24.488	7.00

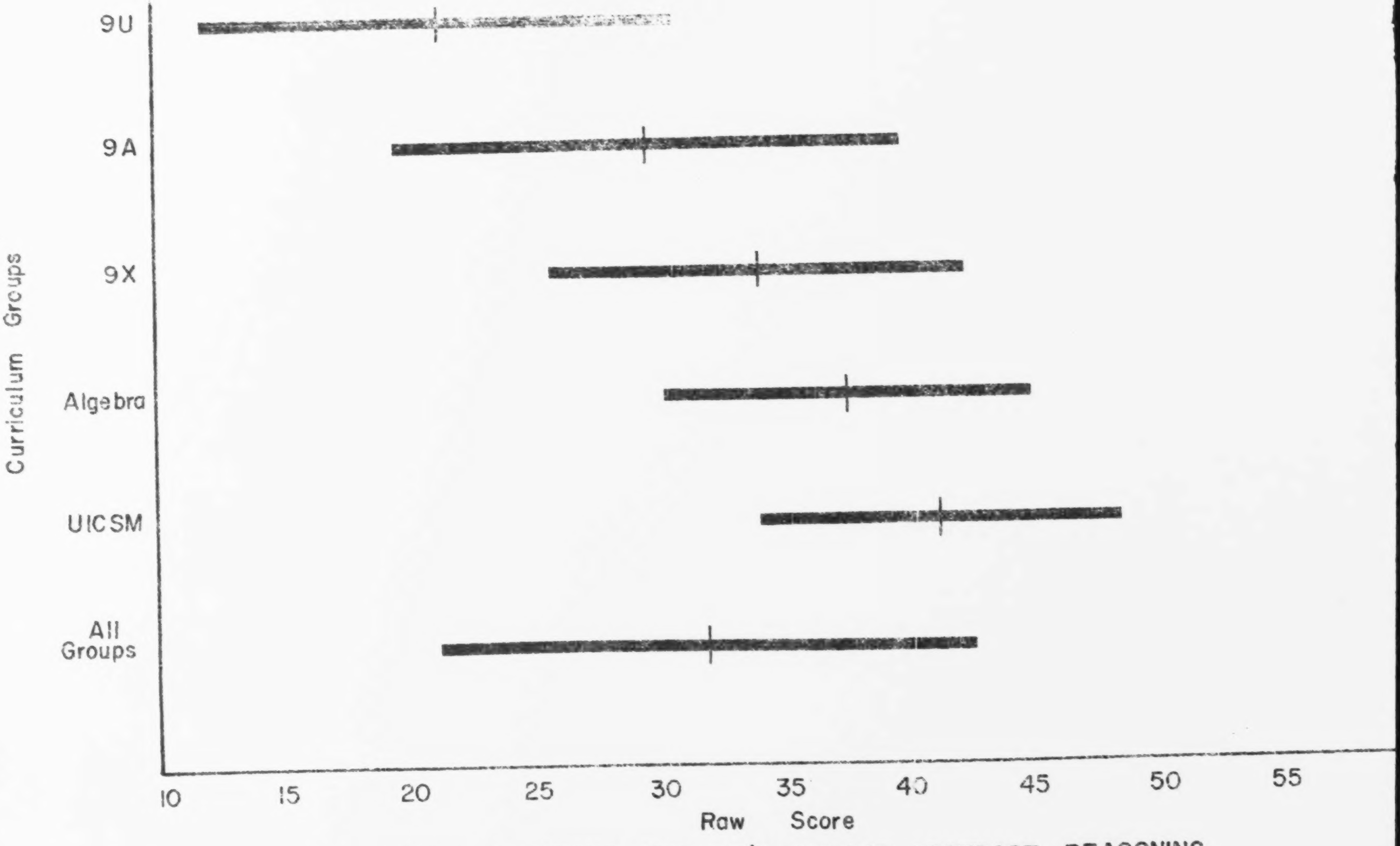
Table 10

Summary Statistics for the Differential Aptitude Test

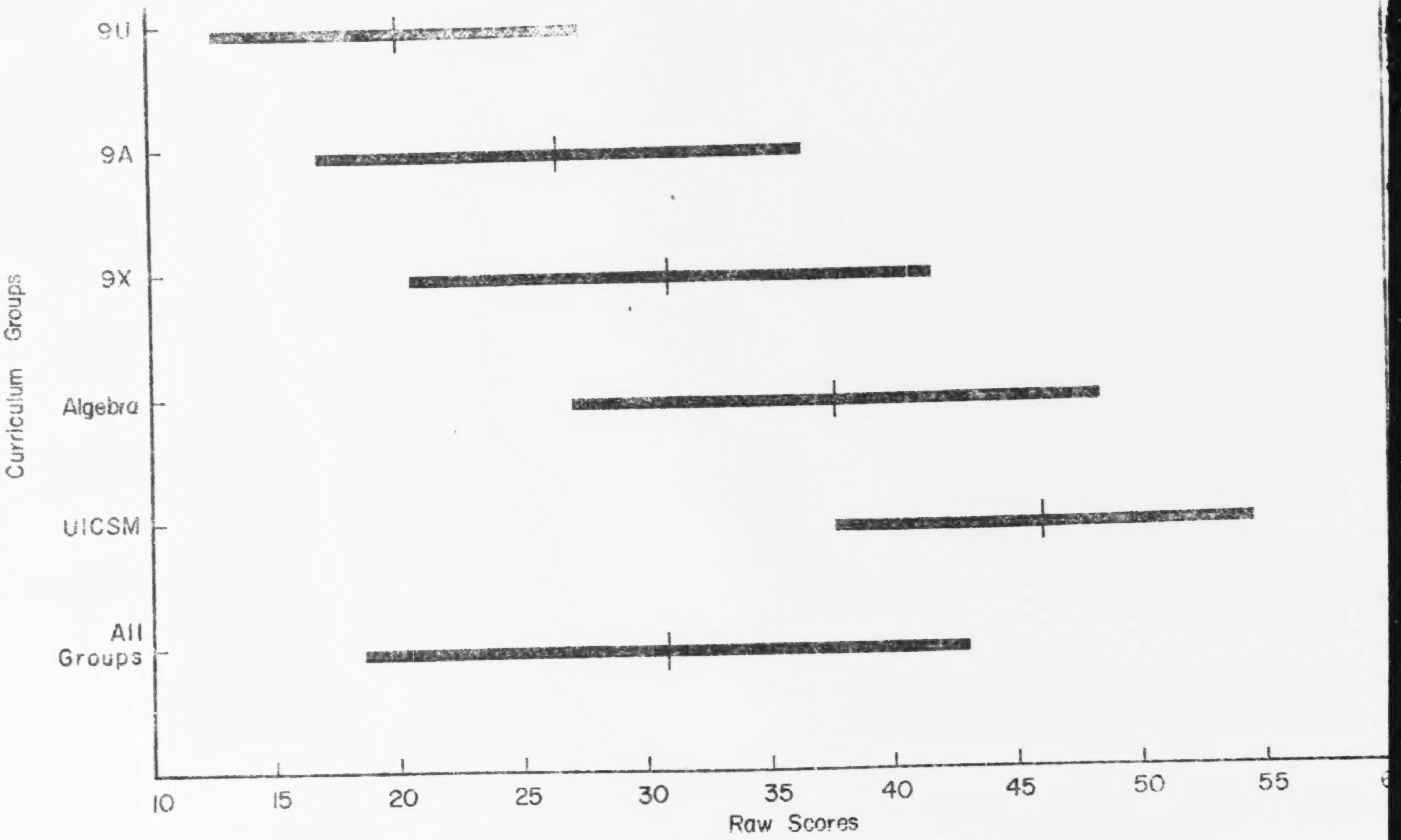
Subtest	Total			Boys			Girls		
	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.	N	Raw Score Mean	S.D.
Verbal Reasoning	473	24.038	10.89	225	24.164	11.12	248	23.923	10.69
Numerical Ability	474	21.236	7.60	226	21.415	7.94	248	21.072	7.30
Abstract Reasoning	473	32.198	10.76	225	32.777	10.58	248	31.673	10.91
Space Relations	475	30.823	12.25	226	33.238	12.83	249	28.630	11.23
Mechanical Reasoning	474	40.487	9.02	226	44.163	8.75	248	37.137	7.91
Clerical Speed & Accuracy	474	48.799	11.88	226	46.119	11.65	248	51.241	11.57
Language Usage: Spelling	475	71.808	16.57	226	67.221	16.86	249	75.971	15.18
Language Usage: Grammar	475	28.736	10.60	226	26.345	10.37	249	30.907	10.36
Verbal Reasoning + Numerical Ability	473	45.266	17.24	225	45.640	18.05	248	44.927	16.50



GRAPH OF MEANS AND S.D.'S OF DAT NUMERICAL ABILITY SUBTEST FOR FIVE CURRICULA GROUPS



GRAPH OF MEANS AND S.D.'S OF DAT ABSTRACT REASONING SUBTEST FOR FIVE CURRICULA GROUPS



GRAPH OF MEANS AND S.D.'S OF DAT SPACE RELATIONS SUBTEST FOR FIVE CURRICULA GROUPS

Table 11

Distribution of Final Grades for 8th Grade Math

Final Grade	Total	UICSM	Algebra	9X	9A	9U
A (4)	7	1	5	0	1	0
B (3)	62	20	31	6	4	1
C (2)	176	13	65	40	40	18
D (1)	199	0	27	44	70	58
F (0)	27	0	1	0	10	16
No Grade	5	0	1	2	1	1
N	476	34	130	92	126	94

Table 12

Summary Statistics for the Final Grade Received in 8th Grade Mathematics

Group	N	Mean Grade	S.D.
Total Group	472	1.6	.83
Boys	225	1.5	.83
Girls	247	1.6	.83
UICSM	34	2.6	.54
Algebra	129	2.0	.79
Mathematics 9X	90	1.5	.61
Mathematics 9A	125	1.3	.70
Mathematics 9U	93	1.0	.64

in verbal and mathematical aptitudes among the five groups. Table 13 indicates the result of the variance analysis of the SCAT Verbal subtest.

Table 13

Analysis of Variance of the SCAT Verbal Scores of the Five Curricular Groups

Variance	D.F.	Sum of Squares	Mean Square	F	Hypothesis
Among Groups	5	50343.00	10068.60	59.45	rejected
Within Groups	870	147332.00	169.34		
Total	875	197675.00	225.91		

The F ratio of 59.4 (5 and 870 d.f.) indicates a significant variation in verbal abilities among the five groups. This would be anticipated since the initial grouping was partly based on the SCAT Total scores. This analysis, however, was preliminary to the covariance analyses. Ideally, to assess the influence of mathematics aptitude on mathematics achievement, it would be desirable for the groups to be initially equal in verbal abilities. Since this does not occur in the natural setting, statistical procedures are employed, i.e., analysis of covariance, to determine the extent of the variation in mathematical traits, independent of the differences in verbal abilities.

Table 14 presents the t tests for the significance of group differences

Table 14

Comparison of Performance on the SCAT Verbal Subtest by Curricular Groups

Groups	N_1	N_2	Mean Difference	t	Significance
UICSM vs Algebra	34	124	16.94	4.65	$p < .01$
UICSM vs Mathematics 9X	34	92	23.71	6.55	$p < .01$
UICSM vs Mathematics 9A	34	126	31.73	8.79	$p < .01$
UICSM vs Mathematics 9U	34	91	41.04	11.37	$p < .01$
Algebra vs Mathematics 9X	124	92	6.71	5.21	$p < .01$
Algebra vs Mathematics 9A	124	126	14.79	11.46	$p < .01$
Algebra vs Mathematics 9U	124	91	24.20	19.36	$p < .01$
Mathematics 9X vs Mathematics 9A	92	126	8.02	6.57	$p < .01$
Mathematics 9X vs Mathematics 9U	92	91	17.43	14.29	$p < .01$
Mathematics 9A vs Mathematics 9U	126	91	9.41	7.84	$p < .01$

on the SCAT Verbal subtest. The five adjacent groups differed significantly from each other in their verbal intelligence.

The analysis of variance among the five groups on the STEP Mathematics scores is indicated in the original analysis in table 15. The F value of 85.79 (5 and 870 d.f.) indicates a significant variation among groups. The adjusted analysis, in which the original sums of squares is adjusted for initial differences in verbal ability, yielded an F value of 26.87. Obviously a significant variation in mathematics achievement still remains among the group. The t tests for the group mean differences disclosed that sections differed in their levels of prior mathematics achievement. The t tests for the group mean differences are shown in table 16. Inasmuch as the STEP Mathematics subtest scores were used as one of the criteria for placement the intergroup differences were expected. The original and adjusted mean differences on the Orleans Geometry Prognosis Test are shown in table 17. Again, as anticipated, the variation among the groups

Table 15

Analysis of Variance of STEP Mathematics Scores of the Five Curricular Groups-Original and Adjusted Sums of Squares and Mean Squares

		Original Analysis				Adjusted Analysis		
Variance	DF	Sum of Squares	Mean Square	F	DF	Sum of Squares	Mean Square	F
Among Groups	5	54671.50	10934.30	85.79	5	11104.74	2220.94	26.87
Within Groups	870	110877.00	127.44		869	71810.63	82.63	
Total	875	165548.50	189.19		874	82915.38	94.86	

Table 16

Comparison of Performance on the STEP Mathematics Subtest by Curricular Groups

Groups	N_1	N_2	Mean Difference	t	Significance
UICSM vs Algebra	34	125	11.52	8.59	$p < .01$
UICSM vs Mathematics 9X	34	92	21.21	10.93	$p < .01$
UICSM vs Mathematics 9A	34	126	27.61	20.15	$p < .01$
UICSM vs Mathematics 9U	34	91	39.63	24.93	$p < .01$
Algebra vs Mathematics 9X	125	92	9.70	10.37	$p < .01$
Algebra vs Mathematics 9A	125	126	16.09	16.19	$p < .01$
Algebra vs Mathematics 9U	125	91	28.12	21.97	$p < .01$
Mathematics 9X vs Mathematics 9A	92	126	6.40	6.56	$p < .01$
Mathematics 9X vs Mathematics 9U	92	91	18.42	14.62	$p < .01$
Mathematics 9A vs Mathematics 9U	126	91	12.02	9.18	$p < .01$

Table 17

Analysis of Variance of Orleans Geometry Prognosis Test Scores of The Five Curricular Groups-Original and Adjusted Sums of Squares and Mean Squares

	Original Analysis				Adjusted Analysis			
Variance	DF	Sum of Squares	Mean Square	F	DF	Sum of Squares	Mean Square	F
Among Groups	5	149211.01	29842.20	77.34	5	38375.14	7675.02	26.83
Within Groups	870	335672.70	385.83		869	248531.20	285.99	
Total	875	484883.71	554.15		874	286906.35	328.26	

in the Geometry Prognosis scores is significant. The F values of 77.34 and 26.83 in the two analyses indicated significant intergroup variance. The pattern of group mean differences disclosed by the t tests of significance are presented in table 18. The five groups differed significantly from each other in their aptitudes as measured by the Orleans Geometry Prognosis Test.

Table 18

Comparison of Performance on the Orleans Geometry Prognosis Test by Curricular Groups

Groups	N_1	N_2	Mean Difference	t	Significance
UICSM vs Algebra	33	132	16.79	7.17	$p < .01$
UICSM vs Mathematics 9X	33	91	30.57	11.62	$p < .01$
UICSM vs Mathematics 9A	33	129	50.62	21.72	$p < .01$
UICSM vs Mathematics 9U	33	87	57.25	21.68	$p < .01$
Algebra vs Mathematics 9X	132	91	13.78	6.50	$p < .01$
Algebra vs Mathematics 9A	132	129	33.83	19.54	$p < .01$
Algebra vs Mathematics 9U	132	87	40.46	18.99	$p < .01$
Mathematics 9X vs Mathematics 9A	91	129	20.05	9.52	$p < .01$
Mathematics 9X vs Mathematics 9U	91	87	26.68	10.93	$p < .01$
Mathematics 9A vs Mathematics 9U	129	87	6.63	3.13	$p < .01$

The F ratios for the original and adjusted analyses of variance on the Cooperative Mathematics Test distributions are shown in table 19.

Table 19

Analysis of Variance of Cooperative Mathematics Test Scores of the Five Curricular Groups-Original and Adjusted Sums of Squares and Mean Squares

	Original Analysis				Adjusted Analysis			
Variance	DF	Sum of Squares	Mean Square	F	DF	Sum of Squares	Mean Square	F
Among Groups	5	183964.84	36792.96	103.08	5	40445.53	8089.10	36.90
Within Groups	870	310504.95	356.90		869	190492.65	219.20	
Total	875	494469.79	565.10		874	230938.18	264.23	

The variation in the adjusted sums of squares indicated that the group differences on the CMT, independent of the variation in verbal abilities, were significant. The critical ratios presented in table 20 indicate that the five curricular groups differed significantly from each other with respect to performance on the CMT.

Table 20

Comparison of Performance on the Cooperative Mathematics Test by Curricular Groups

Groups	N_1	N_2	Mean Difference	t	Significance
UICSM vs Algebra	34	128	24.72	6.81	$p < .01$
UICSM vs Mathematics 9X	34	92	45.44	16.70	$p < .01$
UICSM vs Mathematics 9A	34	125	55.36	20.35	$p < .01$
UICSM vs Mathematics 9U	34	93	71.80	26.11	$p < .01$
Algebra vs Mathematics 9X	128	92	20.71	12.59	$p < .01$
Algebra vs Mathematics 9A	128	125	30.63	18.64	$p < .01$
Algebra vs Mathematics 9U	128	93	47.08	27.69	$p < .01$
Mathematics 9X vs Mathematics 9A	92	125	9.92	7.12	$p < .01$
Mathematics 9X vs Mathematics 9U	92	93	26.36	18.06	$p < .01$
Mathematics 9A vs Mathematics 9U	125	93	16.44	14.52	$p < .01$

The variance and covariance of Algebra Prognosis scores are shown in table 21. The adjusted analysis yielded an F value of 49.9, indicating significant variation among the five groups in their algebra aptitude.

Table 21

Analysis of Variance of Orleans Algebra Prognosis Test Scores of the Five Curricular Groups-Original and Adjusted Sums of Squares and Mean Squares

	Original Analysis				Adjusted Analysis			
Variance	DF	Sum of Squares	Mean Square	F	DF	Sum of Squares	Mean Square	F
Among Groups	5	185208.93	37041.78	105.54	5	63702.21	12740.44	49.99
Within Groups	870	305326.71	350.95		869	221443.85	254.82	
Total	875	490535.65	560.61		874	285146.07	326.25	

The t tests for intergroup differences are presented in table 22. The test of the mean difference between the UICSM and algebra groups indicated no significant difference in the algebra aptitude of these two groups. Significant differences appeared, however, between the other sections.

Table 22

Comparison of Performance on the Orleans Algebra Prognosis Test by Curricular Groups

Groups	N_1	N_2	Mean Difference	t	Significance
UICSM vs Algebra	33	132	1.32	.98	$p > .05$
UICSM vs Mathematics 9X	33	91	21.77	13.52	$p < .01$
UICSM vs Mathematics 9A	33	129	36.77	22.41	$p < .01$
UICSM vs Mathematics 9U	33	87	56.06	28.32	$p < .01$
Algebra vs Mathematics 9X	132	91	20.46	14.15	$p < .01$
Algebra vs Mathematics 9A	132	129	35.45	24.03	$p < .01$
Algebra vs Mathematics 9U	132	87	54.75	29.59	$p < .01$
Mathematics 9X vs Mathematics 9A	91	129	14.99	8.69	$p < .01$
Mathematics 9X vs Mathematics 9U	91	87	34.29	16.65	$p < .01$
Mathematics 9A vs Mathematics 9U	129	87	19.30	9.28	$p < .01$

Factor Analyses of Aptitude Test Variables

It was hypothesized that mathematical aptitude is comprised of the following specific abilities: 1) Numerical, 2) Abstract Reasoning, 3) Spatial Relationships, 4) Logical Reasoning, and 5) Symbol Manipulation. There was no intentional effort to obtain tests, nor to design tests that would be factor-pure. Such an approach (e.g., Peterson et al 1965) would insure their representation in the subsequent analysis. The question would still remain, however, as to whether the obtained factors would merely be constructs in a theory of intelligence, or actually represent mathematical aptitudes which could eventuate in mathematics achievement. In this study the decision was made to follow an empirical rather than a theoretical approach. It was decided to determine the structure of a battery of aptitude tests of known predictive validity rather than to use tests for which both the predictive and the factor validities are unknown.

The intercorrelations of the 29 aptitude test variables are shown in table 23. The subtests on the CMT, OAPT, and OGPT were also summed

Table 23

Intercorrelations of Scores from Mathematical Abilities Tests

Variables	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
SCAT Q	95	41	43	30	33	32	42	37	37	32	31	31	37	25	28	33	28	30	35	37	23	28	32	27	32	45	42	41
STEP M		40	42	32	32	30	43	39	36	32	30	30	37	23	27	33	29	31	36	37	22	27	32	27	31	43	42	42
CMT Sk			74	75	75	58	49	42	58	53	56	57	60	62	59	62	52	50	53	58	39	58	51	47	59	74	54	57
CMT Fa				74	70	53	50	41	58	55	61	58	58	61	57	64	54	55	54	61	41	57	53	50	62	69	55	56
CMT Appl					81	45	36	28	46	43	51	55	52	61	56	59	54	52	47	55	38	61	51	52	62	68	52	58
CMT Appr						48	37	30	45	43	50	51	50	56	53	55	52	51	47	53	40	57	50	45	60	66	51	56
OAPT Ar							59	51	62	62	64	59	62	63	63	64	55	57	57	62	41	53	53	45	57	67	47	47
OAPT Su								75	75	73	66	60	67	52	57	67	50	51	62	64	36	43	51	40	54	60	50	49
OAPT Use									69	62	54	53	60	42	50	61	40	43	53	56	29	34	41	30	42	52	47	42
OAPT Me										75	70	64	72	61	66	72	54	59	66	67	38	51	52	46	62	66	53	50
OAPT Su											70	63	65	58	58	68	46	55	59	64	37	46	49	41	55	62	46	46
OAPT Su												70	73	67	70	75	62	66	63	71	45	60	57	50	67	64	52	54
OAPT Re													67	63	64	74	56	57	60	63	42	56	56	50	62	61	47	52
OAPT Po														64	71	73	61	59	66	72	42	57	57	48	65	67	52	55
OAPT Pr															74	70	64	62	59	69	42	66	56	56	68	71	49	54
OAPT Add																75	61	62	63	69	45	63	56	51	69	68	52	53
OAPT Sum																	63	65	67	70	46	61	60	53	70	71	54	58
OGPT Ax																		65	59	65	45	70	65	62	71	58	48	53
OGPT Rdg																			65	66	48	66	59	54	72	59	45	55
OGPT Ki																				69	41	57	57	47	69	59	51	51
OGPT Co																					50	67	64	53	75	70	55	57
OGPT Un																						49	47	42	53	44	33	37
OGPT Bi																							63	65	76	61	45	58
OGPT Ge																								58	70	56	43	51
OGPT Ge																									64	51	35	40
OGPT Su																										66	48	58
DAT N																											67	66
DAT Abs																												63
DAT Sp																												

Note: Decimals omitted

to obtain total scores for each of these tests. The high correlations among the subtests justified this reduction of the matrix of subtests. Tables of the correlations of the 29 subtests in the CMT, OAPT, OGPT, SCAT, and DAT and of the total scores are included in the appendix, tables A through E. The matrices were factor analyzed, using principal components procedures. Various factor representations of the 29-variable matrix were examined, ranging from a seven-factor to a four-factor solution, with orthogonal rotations. Four factors were extracted from the 8-variable matrix and orthogonal and oblique rotations were compared. The unrotated factor matrices are included in tables F & G in the appendix.

The six-factor solution on the 29-variable matrix was selected for interpretation and for the prediction of mathematics achievement. The four-factor solutions of the 29-variable and 8-variable matrices are also compared in the text of this report. The rotated factor matrices for the seven-and five-factor solutions are included in tables H & I in the appendix.

The results of the six-factor solution are shown in table 24. Factor I is identified as a logical reasoning factor in geometry aptitude. It is defined by the Axioms, Reading Angles, Bisection, Geometric Notations, Geometric Problems and Summary Subtests of the Geometry Prognosis Test. The Problems subtest of the Algebra Prognosis Test also received its highest loading on this factor. The minor differences in the loadings for this subtest on factor I (.52), factor IV (.46), and factor III (.43) may be due to error. Since there are only two items on the Problems subtest its capriciousness across factors might be best explained by unreliability.

Table 24

Mathematics Test Variables and Factor Loadings on the Rotated Factor Matrix*

Variables	Factors I	II	III	IV	V	VI	h^2
SCAT Quantitative	120	922	175	-203	-115	-045	953
STEP Mathematics	118	927	150	-196	-125	-034	953
CMT Skills	247	185	762	-364	-133	-072	833
CMT Facts, Terms & Concepts	307	226	691	-358	-119	-090	775
CMT Applications	366	091	810	-166	-177	-057	861
CMT Appreciation	295	113	795	-177	-184	-128	815
OAPT Arithmetic Test	327	073	316	-606	-093	-153	612
OAPT Substitution in Monomials	215	250	099	-816	-142	-078	811
OAPT Use of Exponents	093	234	038	-799	-202	-044	747
OAPT Meaning of Exponents	284	144	236	-789	-117	-049	796
OAPT Substitution in Monomials with Exponents	204	104	256	-799	-028	-102	769
OAPT Substitution in Binomials with Exponents	438	040	269	-659	-147	-108	734
OAPT Representation of Relations	390	060	357	-609	-053	-071	663
OAPT Positive and Negative Numbers	391	124	279	-670	-177	-064	731
OAPT Problems	526	-043	435	-468	-142	-026	709
OAPT Addition of Like Terms	472	-017	340	-558	-200	-084	698
OAPT Summary Test	433	049	348	-658	-173	-081	781
OGPT Axioms	733	079	219	-304	-205	-061	730
OGPT Reading Angles	629	080	195	-392	-229	-173	678
OGPT Kinds of Angles	496	147	156	-553	-234	-082	660
OGPT Complementary and Supplementary Angles	526	126	252	-548	-238	-177	746
OGPT Understanding Geometrical Relationships	334	071	185	-221	-076	-873	969
OGPT Bisection	740	064	341	-214	-171	-132	762
OGPT Geometrical Notation	661	162	201	-317	-128	-178	653
OGPT Geometrical Problems	743	151	268	-196	108	-038	699
OGPT Summary Test	711	084	309	-361	-170	-177	800
DAT Numerical Ability	314	187	548	-488	-342	-103	801
DAT Abstract Reasoning	174	215	340	-362	-702	-064	820
DAT Space Relations	352	205	351	-274	-648	-064	790
Eigenvalues	16,432	1,824	1.675	1.111	.672	.646	
Cumulative Proportion	.566	.629	.637	.725	.748	.771	

*Decimals are omitted

The highest loading on the first factor, however, might be due to several reasons, for example, 1) a slightly higher level of difficulty, 2) the items can be solved through the use of logical reasoning, or 3) the item-contents are geometric progressions. The first factor in the principal axis solution typically has the highest eigenvalue and accounts for the major portion of the variance. Inasmuch as the first factor was associated with more than 50% of the variance it might be interpreted as a general factor. The low loadings from the SCAT Quantitative, STEP Mathematics, DAT Abstract, and some of the Algebra Prognosis subtests oppose a general-factor interpretation, however.

Factor II is defined by the SCAT Quantitative and STEP Mathematics subtests. The original intention for including the STEP Mathematics Achievement subtest in the analyses was that it might serve as a marker variable and identify achievement components in the prognosis subtests. If it functioned in that manner, however, it did so only for the SCAT Quantitative subtest. In view of the high correlation between the two subtests (.954) it seems difficult to reconcile that one is named an ability test and the other an achievement test. The low loadings of the other 27 variables on this factor serve to clarify it as a SCAT-STEP instrument factor.

The third factor is defined by the four subtests of the Mathematics Placement test and the DAT Numerical Aptitude subtest. The secondary loadings from other arithmetic-type subtests aid in identifying this factor as arithmetic aptitude. It might be described more specifically as the ability to learn and to apply principles of arithmetic.

The fourth factor is defined by the subtests in the Algebra Prognosis Test and by two subtests of the Geometry Prognosis Test, Kinds of Angles and Complimentary-Supplementary Angles. In spite of the OGPT variable loadings, it is identified as algebra aptitude. The two geometry subtests involve the symbolic representation of angles. The failure of the DAT Abstract Reasoning variable to receive its primary loading on this factor was contrary to the initial hypotheses. One of the possible reasons is that the algebra aptitude factor is more heavily weighted with symbol-manipulation ability than with abstract reasoning.

The fifth factor is defined by the DAT Abstract Reasoning and Spatial Relationship variables. The correlation between the two subtests of .63 observed in this study is somewhat higher than reported in the test standardization. Similar to factor II, one possible interpretation is that this is also an instrument factor. Adjacent subtests in the same battery often load on the same factor because the subtests are taken sequentially. The loading for the DAT Numerical variable is a tertiary loading so two mathematical aptitudes are actually reflected in the three subtests. In defining the fifth factor it appears more logical to conclude that the Space Relations subtest required abstract reasoning abilities rather than presume that the Abstract Reasoning subtest required spatial abilities. None of the other variables showed secondary loadings on this factor, so its identification resides in these two subtests. Again, the initial hypothesis about abstract reasoning and spatial relationship as independent factors was not supported.

The sixth factor is primarily defined by one subtest in the OGPT, Understanding Geometric Relationships. Although this variable had a secondary loading on the first factor, the sixth factor appears to be a test-specific factor. The Geometry Prognosis subtests were distributed on three of the six factors whereas the other test batteries tended to load on one factor. The point of view that mathematical aptitudes are a heterogeneous set of specific abilities may be more evident in the case of geometry aptitude than for algebra.

The six factors in this solution were associated with 77% of the variance in the matrix. The use of factor scores for the prediction of mathematics achievement is therefore complicated by the fact that 23% of the aptitude variance is residual. The additional factor in the seven-factor solution increased the variance association by less than 2%.

Although the six factors obtained on this analysis were used in the regression analyses this does not represent the most parsimonious solution. Three other analyses were carried out. The correlations of the eight test variables are shown in table 25. A four-factor solution was then obtained for the eight-variable correlation matrix as shown in table 26.

Table 25

Correlations of the 8 Mathematical Aptitude Variables

	STEP M	CMT	OAPT	OGPT	DAT NA	DAT AR	DAT SR
SCAT Q	.9544	.3993	.3919	.3788	.4574	.4254	.4135
STEP M		.3934	.3828	.3803	.4367	.4235	.4247
CMT Total			.6875	.7135	.7433	.5379	.6060
OAPT Total				.8465	.7963	.6066	.6265
OGPT Total					.7299	.5640	.6331
DAT NA						.6787	.6652
DAT AR							.6339
DAT SR							

Table 26

Mathematics Test Variables and Factor Loadings on the Rotated Factor Matrix*

Variables	I	II	III	IV	h^2
SCAT Q	-202	-951	141	102	977
STEP M	-188	-952	131	130	977
CMT Total	-772	-198	342	104	765
OAPT Total	-870	-170	183	225	870
OGPT Total	-879	-169	087	270	883
DAT NA	-749	-234	418	202	833
DAT AR	-374	-215	839	253	955
DAT SR	-428	-210	286	822	987
Eigenvalues	5.033	1.319	.522	.374	
Cumulative Proportion of Total Variance	.629	.794	.859	.906	

*Decimals are omitted

The test-specific factors which appeared for the Cooperative Mathematics and the Orleans tests in the 29 variable analysis were not maintained in this analysis. The first factor in this analysis appears to be a general factor, and was defined by the Cooperative Mathematics, Algebra Prognosis, Geometry Prognosis and the DAT Numerical Abilities tests. The intercorrelations of these tests ranged from .56 to .84. Factor I was associated with 63% of the variance in the matrix.

The second factor, the SCAT Quantitative-STEP Mathematics factor was maintained as in the 29 variable analysis. The moderate correlation between the SCAT Quantitative and the DAT Numerical Abilities tests (r = .45) raises a question about their content. Obviously the traits assessed by the DAT Numerical Abilities and the Cooperative Mathematics Placement tests are essentially different from the traits assessed by the SCAT Quantitative subtest. Inasmuch as any one of these tests may be used for grouping students in mathematics courses the question about their independent variance (80%) represents an important problem for curriculum guidance.

The third factor is defined by the DAT Abstract Reasoning Test. The correlations of the Abstract Reasoning Test with the Cooperative Mathematics and with the Algebra Prognosis Test were both around .60; however, it shared more of the variance in the Cooperative Mathematics Test than in the Algebra Prognosis Test.

The fourth factor is the DAT Space Relations subtest. The correlation of the Geometry Prognosis with the Space Relations test scores was .63; however, the loading for the Geometry Prognosis variable on this factor was only .27. Evidently geometry aptitude, as assessed by the Orleans Prognosis Test at this age level does not involve as much spatial ability as may generally be assumed as essential to geometry. The four factors were correlated with more than 90% of the variance in the eight-variable correlation matrix.

Two other four-factor solutions were obtained for the 29 variable matrix. The first solution was a principal components analysis with an orthogonal rotation using varimax procedures. The second solution was also a principal components analysis with an oblique rotation, using Harris & Kaiser's (1964) orthogonal transformation procedures.

The varimax rotation of the four principal axes is shown in table 27. The first factor is the algebra aptitude factor, defined primarily by the Orleans Algebra Prognosis Test. One of the Geometry Prognosis subtests, Kind of Angles, also showed a primary loading on this factor.

Mathematics Test Variables and Loadings on the Rotated Factor Matrix*

Variables	Factors I	II	III	IV	h^2
SCAT Quantitative	203	909	190	-127	920
STEP Mathematics	203	934	174	-127	960
CMT Skills	346	177	725	-294	765
CMT Facts and Terms	342	211	641	-357	701
CMT Applications	151	095	800	-390	825
CMT Appreciation	174	122	761	-352	748
OAPT Arithmetic Test	555	096	310	-398	572
OAPT Substitution in Monomials	791	238	137	-255	767
OAPT Use of Exponents	750	225	108	-147	647
OAPT Meaning of Exponents	757	144	253	-323	763
OAPT Substitution in Monomials with Exponents	739	104	245	-273	692
OAPT Substitution in Binomials with Exponents	627	068	285	-488	719
OAPT Representation of Relations	560	075	340	-430	621
OAPT Positive and Negative Numbers	639	140	305	-430	707
OAPT Problems	446	000	427	-544	678
OAPT Addition of Like Terms	535	031	360	-513	681
OAPT Summary Test	634	073	364	-476	766
OGPT Axioms	311	100	271	-701	672
OGPT Reading Angles	392	112	254	-641	643
OGPT Kinds of Angles	533	163	225	-507	619
OGPT Complementary and Supplementary Angles	535	152	295	-578	731
OGPT Understanding Relationships	237	094	206	-487	345
OGPT Bisection	212	088	359	-750	746
OGPT Geometrical Notation	321	156	246	-643	602
OGPT Geometrical Problems	200	108	254	-645	532
OGPT Summary Test	345	106	326	-756	809
DAT Numerical Ability	493	213	585	-370	768
DAT Abstract Reasoning	407	265	446	-249	498
DAT Spatial Relationships	334	252	457	-382	531
Eigenvalues	16.433	1.819	1.679	1.111	

*Decimals are omitted

Variables which showed high secondary loadings were the Complementary and Supplementary Angles subtest in the OGPT and the Numerical, Abstract Reasoning and Spatial Relations subtests of the DAT.

The second factor which consistently appeared in the various analyses was the SCAT Quantitative-STEP Mathematics factor. The third factor was defined by the Cooperative Mathematics Placement subtests and the DAT Numerical, Abstract Reasoning and Spatial Relationship variables. This was identified as the arithmetic aptitude factor in the six-factor solution.

The fourth factor is identified as the geometry aptitude factor and is defined primarily by the Geometry Prognosis subtests and by the Problems subtest of the Algebra Prognosis Test. This factor appears as the first factor in the six-factor solution. A major difference in the four-factor solutions for the 29-and 8-variable matrices appears in the fourth factor. In the 29 variable analysis the Spatial Relations subtest loads on the third factor and is interpreted as an aspect of arithmetic aptitude. In the 8-variable matrix this subtest identifies the fourth factor and is interpreted as a specific aptitude.

The variable loadings for the oblique rotation of the four factors are shown in table 28. The variable loadings on the four factors are generally lower. A slight change also appeared in the primary loading for the Complementary and Supplementary Angles subtest. The factor correlations are shown in table 29.

The transformation matrix for the oblique rotation is shown in table 30. As indicated in table 29 three of the factors are highly correlated, I with III, I with IV and III with IV, whereas factor II is

Table 28

Oblique Rotation of Four Mathematics Aptitude Factors*

Variables	Factors I	 II	 III	 IV
SCAT Quantitative	-056	818	065	-051
STEP Mathematics	-058	846	049	-058
CMT Skills	052	-030	520	091
CMT Facts, Terms & Concepts	041	020	421	006
CMT Applications	-154	-117	598	-033
CMT Appreciation	-118	-081	567	-007
OAPT Arithmetic Test	310	-030	068	-043
OAPT Substitution in Monomials	577	135	-095	102
OAPT Use of Exponents	580	136	-082	172
OAPT Meaning of Exponents	521	017	000	075
OAPT Substitution in Monomials with Exponents	528	-016	013	113
OAPT Substitution in Binomials with Exponents	360	-059	006	099
OAPT Representation of Relations	304	-059	086	-057
OAPT Positive and Negative Numbers	372	006	037	-044
OAPT Problems	162	-148	154	-165
OAPT Addition of Like Terms	260	-107	085	-128
OAPT Summary Test	353	-073	081	-061
OGPT Axioms	004	-009	-019	-410
OGPT Reading Angles	098	001	-032	-334
OGPT Kinds of Angles	264	052	-043	-178
OGPT Complementary and Supplementary Angles	232	023	-000	-215
OGPT Understanding Geometrical Relationships	014	010	-001	-274
OGPT Bisection	-114	-037	064	-461
OGPT Geometrical Notation	026	050	-030	-369
OGPT Geometrical Problems	-077	010	000	-410
OGPT Summary Test	007	-021	007	-428
DAT Numerical Ability	186	023	339	034
DAT Abstract Reasoning	161	117	259	057
DAT Space Relations	055	101	242	-080

*Decimals are omitted

only slightly correlated with the others. The oblique rotations yielded

Table 29

Correlations of the Oblique Axes

Factors	II	III	IV
I	.37	.76	-.82
II		.40	-.29
III			-.81

Table 30

Transformation Matrix for the Oblique Rotation*

Factors	I	II	III	IV
I	237	044	156	-151
II	044	890	051	306
III	-875	054	670	-224
IV	418	450	723	912

*Decimals omitted

essentially the same information as the orthogonal solutions.

The correlation of aptitude factors may have some meaning for the theory of mathematical abilities. If the specific mathematical aptitudes develop unevenly during the course of adolescent mental development, then the trait correlations would be expected. Longitudinal research will be necessary, however, to obtain more conclusive data about the development of mathematical abilities. Canonical correlations of factor matrices at succeeding age levels may also provide some information on this problem. The latter approach, however, would require comparable groups at each age level.

Although the various factor analyses in this study yielded consistent results the differences between obtained and hypothesized factors requires further discussion. The appearance of instrument factors, e.g., SCAT Quantitative and STEP Mathematics Achievement, may have been partly due to their adjacency in the administration sequence. The time intervals which occurred between administrations of the SCAT-STEP, the OAPT-OGPT, and the DAT would tend to produce more independence of such results, whereas the adjacency of subtests within batteries would influence their higher intercorrelation. The alternation of subtests from several batteries, although an inconvenient procedure, may need to be employed to minimize the appearance of instrument factors.

Because of the delineation of test batteries in the factor structure none of the factors were interpreted as group factors. The group factors which appeared in Oldham's (1938) and Wrigley's (1958) analyses may have been due to the inclusion of performance tests with ability tests. It is not unreasonable that performance in a later course, e.g., geometry, requires the application of prior mathematics achievements; but the performance should then be identified as composite performance rather than geometry achievement. The questions of general factors vs. specific and group factors, and the correlations of factors in mathematical aptitude need further study, since the answers to such questions can have important implications for the structuring of mathematics curricula.

Multiple Regression Analyses

Multiple regression analyses were performed to predict each of the various criteria: teacher assigned stanine scores, final grades in ninth grade mathematics, and achievement test scores for each of the five curricular groups. The means and standard deviations were computed for the criterion measures. These statistics are shown in tables 31 through 35. Final grades and stanine scores for the total class are in table K of the appendix. Regression analyses were first computed using the 29 mathematical aptitude and achievement variables as predictors.

Table 31

Summary Statistics of Criterion Measures For UICSM Curricular Group

	Total			Boys			Girls		
Criterion	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Final Grade	33	2.6	.78	19	2.6	.68	14	2.6	.92
Stanine Score	33	5.0	1.89	19	5.0	1.87	14	5.1	1.99
UICSM Final Exam	29	18.2	3.18	17	18.4	3.60	12	18.0	2.40

Table 32

Summary Statistics of Criterion Measures For the Algebra Curricular Group

	Total			Boys			Girls		
Criterion	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Final Grade	130	2.2	.87	66	2.2	.87	64	2.1	.88
Stanine Score	130	4.9	1.87	66	4.8	1.76	64	5.0	1.99
Cooperative Algebra Test Raw Scores	130	31.3	5.60	66	31.7	4.78	64	30.9	6.36

Table 33

Summary Statistics of Criterion Measures
for Mathematics 9X

Criterion	Total			Boys			Girls		
	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Final Grade	93	2.5	.95	38	2.6	.88	55	2.4	.99
Stanine Score	91	5.0	1.85	37	4.8	1.77	54	5.2	1.89
SRA Arithmetic Achievement Test-Part 1	88	29.7	6.82	36	30.2	6.59	52	29.4	7.02
SRA Part 2	87	24.8	3.64	36	25.5	3.18	51	24.3	3.90
SRA Part 3	87	38.4	5.71	36	38.5	4.27	51	38.3	6.58
SRA Total	87	93.0	13.23	36	94.2	10.71	51	92.1	14.79

Table 34

Summary Statistics of Criterion Measures
for Mathematics 9A

Criterion	Total			Boys			Girls		
	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Final Grade	123	1.8	.92	50	1.8	.96	73	1.8	.90
Stanine Scores	127	5.1	1.88	54	4.9	1.97	73	5.2	1.81
SRA Arithmetic Achievement Test-Part 1	120	24.1	7.16	50	25.2	8.03	70	23.2	6.39
SRA Part 2	122	20.8	5.28	51	21.1	5.85	71	20.6	4.87
SRA Part 3	122	35.4	7.85	51	34.8	9.53	71	35.9	6.40
SRA Total	120	80.3	16.74	50	81.0	19.65	70	79.9	14.45

Table 35

Summary Statistics of Criterion Measures
for Mathematics 9U

Criterion	Total			Boys			Girls		
	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Final Grade	83	1.4	.85	43	1.5	.82	40	1.2	.87
Stanine Score	86	4.9	2.09	48	5.0	2.16	38	4.7	2.02
SRA Arithmetic Achievement Test-Part 1	78	17.7	9.54	45	18.5	11.43	33	16.6	6.09
SRA Part 2	78	14.7	5.21	45	15.4	5.25	33	13.9	5.13
SRA Part 3	78	24.5	8.59	45	24.5	8.83	33	24.6	8.40
SRA Total	78	56.4	17.41	45	57.0	17.47	33	55.5	17.55

Regression equations were then obtained from stepwise multiple regression analyses of the same 29 independent variables. Following this, multiple regression analyses were computed using the 6 factor scores for the independent variables; and stanines, grades and achievement test scores for dependent variables. To test the hypothesis that specific mathematical aptitudes have greater predictive validity for achievement than a test of general scholastic aptitude, regression analyses were carried out to predict each of the dependent variables from the SCAT Total scores.

Predictions of Stanine Scores

The correlations of the mathematical aptitude tests and teacher assigned stanines for the five curricular groups are shown in table 36.

Stepwise multiple regression analyses were performed to predict teacher-assigned stanine scores, final grades in ninth grade mathematics and achievement test scores for each of the five curricular groups.

The equation for the prediction of stanines for the UICSM class was

$$Y = .10X_1 + .16X_{16} - 31.11$$

in which X_1 is the SCAT Quantitative subtest, and X_{16} is the ninth subtest of the Orleans Algebra Prognosis Test--addition of like terms. The multiple correlation coefficient of predicted and actual values for the UICSM groups was .99; and 99 per cent of the variance in the stanine scores was accounted for in the analysis. The high correlation could have occurred because of contamination of the teacher-assigned stanines by the test scores, and because of the smaller sample. The SCAT Quantitative subtest accounted for 31 per cent of the variance in the stanines.

Table 36

Zero-Order Correlation Coefficients of 29 Mathematical Aptitude Scores with the Teacher Assigned Stanine Scores for Each of the Five Curricular Groups

Variables	UICSM	Algebra	Mathematics 9X	Mathematics 9A	Mathematics 9U
SCAT Quantitative	56	23	18	39	29
STEP Mathematics	53	20	19	29	22
CMT Skills	39	29	27	35	54
CMT Facts, Terms & Concepts	12	08	09	29	23
CMT Applications	34	17	09	24	12
CMT Appreciation	37	14	33	36	18
OAPT Arithmetic Test	30	26	31	26	44
OAPT Substitution in Monomials	-18	28	29	36	45
OAPT Use of Exponents	-08	-11	16	22	44
OAPT Meaning of Exponents	19	18	29	37	44
OAPT Substitution in Monomials with Exponents	-02	40	37	32	35
OAPT Substitution in Binomials with Exponents	41	33	31	24	39
OAPT Representation of Relations	25	07	19	24	18
OAPT Positive and Negative Numbers	10	39	27	28	38
OAPT Problems	-00	18	42	37	19
OAPT Addition of Like Terms	36	33	24	22	34
OAPT Summary Test	31	33	25	32	40
OGPT Axioms	31	17	30	10	20
OGPT Reading Angles	18	16	39	24	02
OGPT Kinds of Angles	44	24	26	22	01
OGPT Complementary and Supplementary Angles	13	41	29	25	14
OGPT Understanding Relationships	23	18	17	16	10
OGPT Bisection	29	32	27	19	29
OGPT Geometrical Notation	42	19	32	12	10
OGPT Geometrical Problems	34	25	07	22	12
OGPT Summary Test	48	38	24	28	14
DAT Numerical Ability	52	38	44	55	54
DAT Abstract Reasoning	29	08	06	28	50
DAT Spatial Relationships	34	17	20	18	27

Note: Decimals are omitted

The F value from the analysis of variance for the multiple linear regression was 14.69. (d.f.: 29, 3; $.01 < p < .05$)

The stepwise multiple regression equation for the prediction of stanines for the algebra group was

$$Y = -2.24X_9 + .49X_{11} + .64X_{14} + .29X_{17} + .37X_{21} + 5.07$$

in which X_9 is the OAPT Use of Exponents, X_{11} is the OAPT Substitution in Monomials with Exponents, X_{14} is the OAPT Positive and Negative Numbers, X_{17} is the OAPT Summary test and X_{21} is the OGPT Complementary and Supplementary Angles. Each of these variables is included in Factor 4 of the factor analysis. The multiple correlation coefficient of predicted and observed stanine scores from the 29 predictor variables was .72; and 52 per cent of the variance was accounted for by the regression. The OGPT subtest on Complementary and Supplementary Angles contributed 17 per cent to the variance. The analysis of variance for the multiple regression yielded an F value of 3.47 indicating significant linearity. (d.f.: 29, 90; $p < .01$)

Prediction of the teacher assigned stanines in the stepwise multiple regression for Mathematics 9X students resulted in the following equation

$$Y = .08X_3 + .31X_{11} + .08X_{15} + .16X_{19} + .08X_{27} - .82$$

in which X_3 is the CMT Skills subtest, X_{11} is the OAPT Substitution in Monomials, X_{15} is the OAPT Problems, X_{19} is the OGPT Reading Angles and X_{27} is the DAT Numerical Ability. The multiple correlation coefficient was .71 resulting in a coefficient of determination of .51. The DAT

Numerical Ability subtest accounts for 20 per cent of the variance. The F value from the analysis of variance for multiple linear regression was 2.18, indicating significant variation due to linear regression. (d.f. 29, 60; $p < .01$)

The stepwise multiple regression equation predicting stanines for the Mathematics 9A students was

$$Y = .11X_6 + .23X_8 + .15X_{27} + .23$$

in which X_6 is the CMT Appreciations subtest, X_8 is the OAPT Substitution in Monomials, and X_{27} is the DAT Numerical Abilities. The multiple correlation coefficient of actual and predicted values was .71, and the coefficient of determination was .51. The DAT Numerical Abilities subtest accounted for 30 per cent of the variance. The analysis of variance for the multiple linear regression yielded an F value of 3.25 indicating significant variation due to linear regression. (d.f.: 29, 90; $p < .01$)

For the prediction of stanines for Mathematics 9U students the stepwise multiple regression equation was

$$Y = .13X_3 + .44X_{23} - .40X_{24} + .14X_{27} + .05X_{28} + 1.74$$

in which X_3 is the CMT Skills subtest, X_{23} is the OGPT Bisection, X_{24} is the OGPT Geometrical Notation, X_{27} is the DAT Numerical Abilities and X_{28} is the DAT Abstract Reasoning. The multiple correlation coefficient was .79, and 63 per cent of the variance was accounted for in the analysis. The DAT Numerical Abilities subtest accounted for 30 per cent of the variance. The F value was 2.73. (d.f.: 29, 45; $p < .01$)

The predictor variables which were selected in each of the stepwise regression analyses are shown in table 37.

Table 37

Prediction of Teacher Assigned Stanines For Each Curricular Group

Group	Criteria	Stepwise Predictor Variables
UICSM	Stanine Scores	SCAT Quantitative OAPT Addition of Like Terms
Algebra	Stanine Scores	OAPT Use of Exponents OAPT Substitution in Monomials with Exponents OAPT Positive & Negative Integers OAPT Summary Test OGPT Complementary & Supplementary Angles
Mathematics 9X	Stanine Scores	CMT Skills OAPT Substitution in Monomials with Exponents OAPT Problems OGPT Reading Angles DAT Numerical Ability
Mathematics 9A	Stanine Scores	CMT Appreciation OAPT Substitution in Monomials DAT Numerical Ability
Mathematics 9U	Stanine Scores	CMT Skills OGPT Bisection OGPT Geometrical Notation DAT Numerical Ability DAT Abstract Reasoning

Prediction of Achievement Test Scores

The correlations of the mathematical aptitude scores with the achievement test criteria for the five curricula groups are presented in table 38.

Multiple regression analyses were again carried out, using the appropriate achievement tests as criterion measures. The stepwise multiple regression equation predicting achievement on the UICSM final examination was

$$Y = 1.66X_{22} + .62$$

where X_{22} is the OGPT Understanding Geometrical Relationships subtest. This subtest defines the sixth factor in the correlation analysis of the predictor variables. The multiple correlation coefficient from the analysis of the 29 aptitude scores was .96, and the coefficient of determination was .92. Nine per cent of the variance was accounted for by the OGPT subtest. The F value was 1.64. (d.f.: 29, 3; $p > .05$) The equation predicting scores on the Cooperative Algebra Test for the algebra group was

$$Y = 2.27X_{11} + 1.39X_{14} + 1.09X_{17} + .48X_{19} + .85X_{21} - 10.65$$

in which X_{11} is the OAPT Substitution in Monomials with Exponents, X_{14} is the OAPT Positive and Negative numbers, X_{17} is the OAPT Summary test, X_{19} is the OGPT Reading Angles, and X_{21} is the OGPT Complementary and Supplementary Angles. The multiple correlation coefficient of the

Table 38

Zero Order Correlation Coefficients of 29 Mathematical Aptitude Scores with Achievement Test Scores for Each of the Five Curricular Groups

Variables	UICSM Final Exam	Cooperative Algebra Test	SRA Arithmetic Achievement 9X	SRA Arithmetic Achievement 9A	SRA Arithmetic Achievement 9U
SCAT Quantitative	19	18	03	37	40
STEP Mathematics	-06	30	10	32	32
CMT Skills	09	24	17	30	52
CMT Facts, Terms & Concepts	-07	20	-03	31	19
CMT Applications	12	27	18	36	22
CMT Appreciation	-13	23	30	27	21
OAPT Arithmetic Test	15	33	15	26	31
OAPT Substitution in Monomials	-18	36	03	29	43
OAPT Use of Exponents	-10	01	28	15	38
OAPT Meaning of Exponents	-18	28	-08	31	46
OAPT Substitution in Monomials with Exponents	-03	52	-01	35	21
OAPT Substitution in Binomials with Exponents	04	29	08	19	30
OAPT Representation of Relations	20	11	14	09	17
OAPT Positive and Negative Numbers	-07	35	24	20	35
OAPT Problems	-01	24	22	35	22
OAPT Addition of Like Terms	00	33	18	16	38
OAPT Summary Test	21	39	-04	32	30
OGPT Axioms	-13	30	11	11	30
OGPT Reading Angles	07	34	18	21	08
OGPT Kinds of Angles	-00	27	-00	19	09
OGPT Complementary and Supplementary Angles	-02	45	09	25	14
OGPT Understanding Relationships	31	13	-22	16	11
OGPT Bisection	18	34	14	22	22
OGPT Geometrical Notation	19	26	13	09	17
OGPT Geometrical Problems	-18	31	00	27	28
OGPT Summary Test	26	40	07	32	14
DAT Numerical Ability	22	39	24	45	48
DAT Abstract Reasoning	-02	15	00	33	53
DAT Spatial Relationships	10	29	10	10	23

Decimals are omitted

predicted and actual values was .75, and 56 per cent of the variance was accounted for in the analyses. The OAPT Substitution in Monomials with Exponents subtest accounted for 27 per cent of the variance. The F value from the analysis of variance for the multiple linear regression was 4.08 indicating significant variation due to linear regression. (d.f.: 29, 90; $p < .01$)

The Mathematics Achievement Test for grades 6-9, Form A, was used as the test criterion in the 9X, 9A, and 9U groups. The stepwise multiple regression equation for Mathematics 9X was

$$Y = 2.34X_6 + 8.93X_9 - 1.19X_{22} + 27.76$$

in which X_6 is the CMT Appreciation subtest, X_9 is the OAPT Use of Exponents subtest, and X_{22} is the OGPT Understanding Geometrical Relationships. The multiple correlation coefficient was .67 and the coefficient of determination was .45. Nine per cent of the variance was accounted for by the CMT Appreciation subtest. The analysis of variance for the multiple linear regression yielded an F value of 1.7026. (d.f.: 29, 60; $.01 < p < .05$)

The regression equation for Mathematics 9A, predicting SRA Achievement test scores was

$$Y = 3.40X_{17} + 2.04X_{27} + 25.62$$

in which X_{17} is the OAPT Summary test and X_{27} is the DAT Numerical Abilities subtest. The multiple correlation coefficient was .64, accounting for the 41 per cent of the variance in the analysis. The DAT Numerical Abilities subtest accounted for 20 per cent of the variance. An

F value of 2.18 was obtained from the analysis of variance due to regression. (d.f.: 29, 90; $p <.01$)

The regression equation for the Mathematics 9U students was

$$Y = .37X_2 + 1.40X_3 - 2.71X_{24} + 1.65X_{25} + 1.04X_{28} - 74.68$$

in which X_2 is the STEP Mathematics test, X_3 is the CMT Skills subtest, X_{24} is the OGPT Geometrical Rotation subtest and X_{28} is the DAT Abstract Reasoning. The multiple correlation coefficient of the predicted and actual values was .78 and the coefficient of determination was .61. The DAT Abstract Reasoning accounted for 28 per cent of the variance. The F value was 2.52. (d.f.: 29, 45; $p < .01$)

The predictor variables which were selected in the stepwise analyses of achievement test scores for each curricular group are shown in table 39.

The correlations of the aptitude variables with the final course grades for the five curricular groups are shown in table 40.

Prediction of Final Grades

The third criterion of achievement was the final grades from the mathematics classes. The stepwise multiple regression equation for the UICSM class was

$$Y = .11X_{27} - 1.02$$

where X_{27} is the DAT Numerical Abilities test. The multiple correlation coefficient of predicted and actual grades was .99, and the coefficient of determination was .98. Again, group size and contamination of criterion by predictor tests may be involved in this correlation. Forty

Table 39

Prediction of Achievement Tests for Each Curricular Group

Group	Criterion	Stepwise Predictor Variables
UICSM	UICSM Final Exam	Understanding Geometrical Relationships
Algebra	Cooperative Algebra Test Level 1, Form A	OAPT Substitution in Monomials with Exponents OAPT Positive and Negative Numbers OAPT Summary Test OGPT Reading Angles OGPT Complementary and Supplementary Angles
Mathematics 9X	SRA Arithmetic Achievement Test for Grades 6-9, Form A	CMT Appreciation OAPT Use of Exponents OGPT Understanding Geometrical Relationships
Mathematics 9A	SRA Arithmetic Achievement Test for Grades 6-9, Form A	OAPT Summary Test DAT Numerical Ability
Mathematics 9U	SRA Arithmetic Achievement Test for Grades 6-9, Form A	STEP Mathematics CMT Skills OGPT Geometrical Notation OGPT Geometrical Problems DAT Abstract Reasoning

Table 40

Zero Order Correlation Coefficients of 29 Mathematical Aptitude Scores with Final Grades for Each of the Five Curricular Groups

Variables	UICSM	Algebra	Mathematics 9X	Mathematics 9A	Mathematics 9U
SCAT Quantitative	30	26	20	38	33
STEP Mathematics	40	25	25	23	22
CMT Skills	40	25	22	31	53
CMT Facts, Terms & Concepts	09	12	-00	24	37
CMT Applications	24	25	25	28	19
CMT Appreciation	29	18	31	27	33
OAPT Arithmetic Test	35	23	26	21	44
OAPT Substitution in Monomials	-08	19	24	27	48
OAPT Use of Exponents	-08	-07	22	09	49
OAPT Meaning of Exponents	18	20	19	28	42
OAPT Substitution in Monomials with Exponents	-00	41	35	27	37
OAPT Substitution in Binomials with Exponents	24	28	23	27	38
OAPT Representation of Relations	08	16	30	20	23
OAPT Positive and Negative Numbers	14	33	38	30	32
OAPT Problems	10	19	34	25	16
OAPT Addition of Like Terms	36	33	23	31	30
OAPT Summary Test	36	35	24	36	44
OGPT Axioms	14	27	33	03	22
OGPT Reading Angles	04	15	44	23	05
OGPT Kinds of Angles	26	21	40	15	11
OGPT Complementary and Supplementary Angles	15	43	32	12	15
OGPT Understanding Relationships	07	22	23	13	06
OGPT Bisection	24	32	34	20	25
OGPT Geometrical Notation	38	24	29	10	21
OGPT Geometrical Problems	36	28	14	20	15
OGPT Summary Test	35	46	29	21	12
DAT Numerical Ability	63	36	41	41	57
DAT Abstract Reasoning	22	16	14	29	55
DAT Spatial Relationships	30	20	19	07	34

Decimals are omitted

per cent of the variance was accounted for by the DAT Numerical Abilities test. The F value from the analysis of variance for the regression was 6.58. (d.f.: 29, 3; $p > .05$)

The stepwise regression equation for the algebra group, predicting final grades was

$$Y = .25X_{11} + .21X_{14} + .14X_{17} + .10X_{21} + .09X_{26} - 3.55$$

in which X_{11} is the OAPT Substitution in Monomials with Exponents, X_{14} is the OAPT Positive and Negative numbers, X_{17} is the OAPT Summary test, X_{21} is the OGPT Complementary and Supplementary Angles, X_{26} is the OGPT Summary test. The multiple correlation coefficient was .69, and 48 per cent of the variance was accounted for in the analysis. The Summary tes of the OGPT accounted for 21 per cent of the total variance. The F value was 2.97 indicating significant variation due to linear regression. (d.f.: 29, 90; $p < .01$)

For the Mathematics 9X curricular group the stepwise multiple regression equation predicting final grades was

$$Y = .04X_3 + .16X_{11} + .12X_{14} + .13X_{19} - .30$$

in which X_3 is the CMT Skills subtest, X_{11} is the OAPT Substitution in Monomials with Exponents, X_{14} is the OAPT Positive and Negative numbers, and X_{19} is the OGPT Reading Angles. The multiple correlation coefficient of predicted and actual grades was .74, and the coefficient of determination was .55. The analysis of variance for linear regression yielded an F value of 2.55 indicating linearity. (d.f.: 29, 60; $p < .01$)

The stepwise regression equation predicting final grades for

Mathematics 9A was

$$Y = .02X_1 + .14X_{17} + .04X_{27} - 5.59$$

in which X_1 is the SCAT Quantitative, X_{17} is the Summary test of the OAPT and X_{27} is the DAT Numerical Abilities test. The multiple correlation coefficient obtained from the analysis of the 29 variables against the criterion variable was .65. The F value was 2.28 indicating significant variation due to regression. (d.f.: 29, 90; $p < .01$) In this analysis 42 per cent of the criterion variance was accounted for in the analysis and 17 per cent of the variance was accounted for by the DAT Numerical Abilities.

In predicting final grades for Mathematics 9U students, the stepwise multiple regression equation was

$$Y = .04X_3 + .06X_{27} + .03X_{28} - .36$$

in which X_3 is the CMT Skills subtest, X_{27} is the DAT Numerical Abilities and X_{28} is the DAT Abstract Reasoning. The multiple correlation coefficient of predicted and observed grades was .79, and the coefficient of determination was .62. The DAT Numerical Abilities subtest accounted for 32 per cent of the variance. The F value from the analysis of variance for the multiple linear regression was 2.62 indicating linearity. (d.f.: 29, 45; $p < .01$)

The predictor variables which were selected in the stepwise analyses for prediction of final grades is shown in table 41.

Chapter 4 has presented the major statistical findings of the study. The analyses of variance indicated the range of variation in the various

Table 41

Prediction of 9th Grade Final Mathematics Grades For Each of the Curricular Groups

Group	Criterion	Stepwise Predictor Variables
UICSM	Final Grade	DAT Numerical Ability
Algebra	Final Grade	OAPT Substitution in Monomials with Exponents OAPT Positive and Negative Numbers OAPT Summary Test OGPT Complementary and Supplementary Angles OGPT Summary Test
Mathematics 9X	Final Grade	CMT Skills OAPT Substitution in Monomials with Exponents OAPT Positive and Negative Numbers OGPT Reading Angles
Mathematics 9A	Final Grade	SCAT Quantitative OAPT Summary Test DAT Numerical Ability
Mathematics 9U	Final Grade	CMT Skills DAT Numerical Abilities DAT Abstract Reasoning

test score distributions among the five groups. The factor structure of the 29 aptitude subtests and of the eight aptitude test variables were interpreted. Finally, the predictions of success on achievement tests and on grade criteria were computed. A review of the hypotheses under investigation, and a discussion of the findings will be presented in the following chapter.

Chapter 5

Summary and Discussion

The major objectives for this study were as follows: 1. to analyze the nature of mathematical abilities presented by incoming high school students, and 2. to predict their success or failure in various courses in the mathematics curriculum. Subjects for the study were the 1965-66 ninth grade students at R. L. Stevenson School in Honolulu, Hawaii. There was a total of 476 students enrolled in five mathematics courses, described in the appendix. Their placements in these courses were based on scores for the CMT, on grades received for eighth grade mathematics, and on Total scores for the SCAT. Instruments used in this study were the Cooperative Mathematics Placement Test, School and College Ability Test, Mathematics subtest of the Sequential Tests of Educational Progress, Orleans Algebra Prognosis Tests, Orleans Geometry Prognosis Tests, and the Numerical, Abstract Reasoning and Spatial Relations subtests of the Differential Aptitude Test Battery.

Analysis of variance and covariance were used to study the extent of variations in verbal and mathematical aptitudes for the five groups. Analysis of variance disclosed a significant variation in verbal abilities as measured by the SCAT Verbal subtest. Analysis of covariance procedures were then applied to determine the extent of variation in mathematical traits, independent of the differences in verbal abilities. Significant intergroup variations appeared on the CMT, the OAPT, the OGPT, and the STEP Mathematics tests. Graphs of the means and standard deviations of the aptitude tests illustrated the hierarchy of mathematics abilities represented in the five courses.

The hypotheses under investigation in this research are reviewed as follows:

1. Mathematical ability is not a unitary trait, but rather is comprised of a number of aptitudes such as numerical aptitude, abstract reasoning, and space-form perception.

2. Specific mathematical aptitudes will hold greater predictive validity for achievement in various mathematics courses than will a test of general scholastic aptitude.

3. Specific aptitudes such as algebra aptitude and abstract reasoning will show greater predictive validity for algebra achievement than will other aptitude and achievement predictors.

4. Numerical aptitude will hold greater predictive validity for arithmetic achievement than will other aptitude and achievement predictors.

The subtest of the six tests were intercorrelated in a 29-variable matrix. Total scale scores were also correlated in an 8-variable matrix. The 29-variable and the 8-variable matrices were factor analyzed using principal-components solution with orthogonal and oblique factor rotations. A six-factor solution and two four-factor solutions were reported. The six-factor solution was interpreted as follows: I. Logical Reasoning, II. SCAT Quantitative-STEP Mathematics Instrument Factor, III. Arithmetic Aptitude, IV. Algebra Aptitude, V. Abstract Reasoning-Space Relations, VI. Geometric Relationships.

The four-factor analysis of the 8-variable matrix represented a more parsimonious solution; however, it may be less meaningful for the delineation of specific aptitudes. The four factors were identified as follows: I. General Mathematics Aptitude, II. SCAT Quantitative-STEP Mathematics,

III. Abstract Reasoning, IV. Spatial Relations. In the four-factor solutions the arithmetic and algebra factors collapsed on the first factor and the DAT Spatial Relations subtest was separated as a specific factor.

The four-factor solution of the 29 variable matrix yielded quite different results. The first factor was a specific factor, algebra aptitude. The spatial relations variable loaded on the arithmetic factor, and the fourth factor was clearly a geometry factor. The loadings for the DAT Abstract Reasoning and Spatial Relations subtest on the same factor in the six-factor solution, and on different factors in the four-factor solution, create an interpretation difficulty. Obviously the problem lies in how the domain of mathematical aptitudes is defined rather than in the analyses. The factor analyses clearly supported the first hypothesis that mathematical ability is comprised of a number of aptitudes; but the nature of these aptitudes as specific and independent, or as correlated traits is still in question.

Perhaps a better distinction should be made between specific intelligence factors and aptitudes for achievement in school subjects. Inasmuch as these are generally considered to be synonymous they were combined in this study. The subtests in the Orleans Prognosis Tests, however, can also be described as miniature achievement tests. It is presumed that the students' intellectual abilities are applied in the lessons preceding each subtest; and that the lessons are representative of learning demands which the student will later meet in the course. Perhaps the predominance of the prognosis and placement tests in the domain under study led to the identifications of factors as subject aptitudes. The association of these aptitudes with underlying intellectual factors was not as successful as anticipated for the study. Another approach to the problem then would be to use the mathematics prognosis tests as criteria and specific intelligence factors as predictors.

The predictions of success in the mathematics courses were carried out using three different sets of predictors: 1. the 29-aptitude subtests, 2. the six-factor scores, and 3. the SCAT Verbal and SCAT Total scores. Criteria of success in courses included teacher-assigned stanines, end-of-course achievement tests, and course grades. Comparisons of the correlations between predictors and criteria are shown in tables 42, 43 and 44.

Table 42

Multiple Correlation Coefficients Predicting Teacher Assigned Stanines from Various Independent Variables for Each Curricular Group

Group	29-Aptitude Scores	6-Factor Scores	SCAT Verbal	SCAT Total
UICSM	.99	.59	.03	.36
Algebra	.72	.53	.00	.17
Math 9X	.71	.51	.01	.10
Math 9A	.71	.58	.11	.27
Math 9U	.79	.64	.13	.25

Table 43

Multiple Correlation Coefficients Predicting Achievement Test Scores from Various Independent Variables for Each Curricular Group

Group	29-Aptitude Scores	6-Factor Scores	SCAT Verbal	SCAT Total
UICSM	.96	.60	.38	.02
Algebra	.75	.61	.03	.19
Math 9X	.67	.39	.01	.01
Math 9A	.64	.44	.26	.37
Math 9U	.78	.62	.37	.48

Table 44

Multiple Correlation Coefficients Predicting Mathematics Final Grades from Various Independent Variables for Each Curricular Group

Group	29 Aptitude Scores	6 Factor Scores	SCAT Verbal	SCAT Total
UICSM	.99	.54	.10	.27
Algebra	.69	.55	.02	.21
Math 9X	.74	.57	.04	.12
Math 9A	.65	.46	.08	.25
Math 9U	.79	.68	.20	.30

The hypothesis that mathematical aptitude scores are superior to a general aptitude score for the prediction of mathematics achievement was tested using Fisher's Z test for the significance of the difference between correlation coefficients. The coefficients were converted to Z_r's and the ratio of the difference between the Z_r's to its standard error was computed. The results for the comparison of the six-factor prediction and the SCAT Total prediction are shown in Table 45.

Table 45

Comparison of Correlation Coefficients of Six-Factor Scores and SCAT Total Score with End-of-Course Achievement Tests

		6-Factor Scores		SCAT Total Converted Score			
Group	N	r	Fisher's Z_r	r	Fisher's Z_r	t	p
UICSM	33	.60	.69	.02	.02	2.61	< .01
Algebra	120	.61	.71	.19	.19	3.97	< .01
Math 9X	90	.39	.41	.01	.01	2.68	< .01
Math 9A	120	.44	.47	.37	.39	.61	> .05
Math 9U	75	.62	.73	.48	.52	1.27	> .05

The six-factor scores were superior to the test of general scholastic ability for the predictions of course achievement in the UICSM, Algebra and Mathematics 9X groups. Although the differences in coefficients appearing for the 9A and 9U groups favor the six-factor prediction, the differences might be due to chance variations.

The significance of the differences between the correlations of the 29-aptitude scores and the SCAT Total score with the end-of-course tests were also tested. The Z_r and t values for these analyses are shown in Table 46. The mathematical aptitude scores were significantly more effective than the SCAT Total score in predicting mathematics achievement as measured by the end-of-course tests.

Table 46

Comparison of Correlation Coefficients Predicting Ninth Grade Mathematics Achievement from the SCAT Total Converted Score and the 29 Mathematics Aptitude Test Scores

Group	N	29 Aptitude Scores		SCAT Total Converted Score		t
		r	Fisher's Z_r	r	Fisher's Z_r	
UICSM	33	.96	1.95	.03	.02	7.72
Algebra	120	.75	.97	.19	.19	6.00
Math 9X	90	.67	.81	.01	.01	5.33
Math 9A	120	.64	.76	.37	.39	2.84
Math 9U	75	.78	1.05	.48	.52	3.11

It was hypothesized that algebra aptitude and abstract reasoning would show greater predictive validity for algebra achievement than would the other aptitude and achievement predictors. The best predictors of performance on the Cooperative Algebra Test, the criterion for course achievement, were the OAPT Substitution in Monomials (r = .52), and the OGPT Complimentary and Supplementary Angles subtest (r = .45). In the factor analysis of the aptitude variables both of these subtests received their major loading on the algebra aptitude factor. The same subtests also showed the highest correlations with the teacher-assigned stanines (r = .40 and .41 respectively). Although the significance of the difference between these and other correlation coefficients were not tested these data tend to support the hypothesis. The related hypothesis, that abstract reasoning abilities would be an important predictor of algebra success was not supported, however, in the case of the DAT Abstract Reasoning subtest.

The fourth hypothesis was that numerical ability would hold greater predictive validity for arithmetic achievement than would other aptitude and achievement predictors. The data for the two arithmetic sections, Mathematics 9A and Mathematics 9U tend to support this hypothesis. The best single predictor for the teacher-assigned stanines and for the SRA Arithmetic Achievement Test scores was the DAT Numerical Abilities Test, (r = .55 for stanines and r = .45 for SRA Arithmetic). For Mathematics 9U the Numerical Abilities subtest was the best single predictor for the teacher-assigned stanines. The best predictors for SRA Arithmetic scores in the 9U section were the DAT Abstract Reasoning (r = .53), and the DAT Numerical (r = .48). Although the magnitude of these coefficients may not be significantly greater than some of the other coefficients, e.g., SCAT Total, the evidence favors prediction from a specific aptitude test rather than general ability or achievement tests.

Hypotheses about the predictors of success in the UICSM course were not stated. Also, the Mathematics 9X course was a composite course involving basic algebra concepts and fundamental operations with sets of whole numbers and rational numbers. The Mathematics 9X course approached this through a geometrical approach, using concepts of area, and measurement, with application of manipulative skills. Without any prior knowledge of the degrees of emphasis placed on algebraic concepts, geometric relationships, or symbol manipulation in the 9X course any hypothesis about predictors would tend to be simple conjecture. On the premise that the major emphasis in Mathematics 9X was number relations, and to obtain a criterion that would allow comparisons of the 9X, 9A, and 9U sections, the SRA Arithmetic test was used as the criterion.

The problem of selecting a suitable criterion for course achievement is also illustrated in the predictions for the UICSM course. The best predictors for the teacher-assigned stanines were the SCAT Quantitative and the OAPT Addition of Like Terms. The best predictor for the UICSM end-of-course test was OGPT Understanding Geometric Relations subtest. And the best predictor for course grades was the DAT Numerical Abilities test.

The fact that algebra, geometry, and general mathematics concepts are incorporated in the course content of Mathematics 9X is reflected in the subsets of predictors for the various criteria. For the prediction of the SRA Arithmetic scores the CMT Appreciation, the OAGPT Use of Exponents and the OGPT Understanding Geometric Relationships subtest were the most effective predictors. The best predictors of final grades were the CMT Skill test, the OAPT Substitution in Monomials, the OAPT Positive and Negative Numbers, and the OGPT Reading Angles. These subtests, with the addition of the OAPT Problems and the DAT Numerical Ability subtests were selected for the prediction of teacher-assigned stanines. The assumption that Mathematics 9X had a greater emphasis on arithmetic processes and number relations than on algebraic or geometric concepts was not supported in the regression analyses. The stanines and course grades criteria required predictors different from those selected for the arithmetic achievement criterion. This is an indication that the stanines and course grades represent a broader range of achievement than the test criterion. To evaluate achievement in a composite mathematics course such as 9X it would probably be desirable to use tests which sample from the algebra and geometry content as well as from the arithmetic content.

In the past year or two the distinction between modern and traditional mathematics has begun to decline. Perhaps the modern innovations have attained their purpose with the wider recognition of objectives such as conceptual understanding and the knowledge of principles. The organization of the curriculum on dual tracks, i.e., modern vs. traditional courses can be critically questioned. Even before the evaluation of modern mathematics has been completed some of the previously unrecognized values of traditional mathematics have become apparent. The organization of this curriculum appears now to be in a transitional state. Modern mathematics courses, identified by abbreviations for their origin are still appended to the previous curriculum. The term Mathematics is also used to identify grade-level and remedial arithmetic courses, as well as composite courses with varied subject content. It is difficult to discern from the present situation whether subject oriented courses with modernization will prevail, or whether composite mathematics courses which cross subject boundaries will prove to be more popular. Perhaps before the questions pertaining to curriculum organization are resolved further information may be available about the nature of mathematical-learning abilities.

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APPENDIX

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Description of Courses[1]

The ninth grade curriculum at Stevenson Intermediate School included five different mathematics courses. These courses were UICSM, Algebra 1-2, Mathematics 9X, Mathematics 9A and Mathematics 9U.

In the UICSM course emphasis is placed on the structure of mathematics. Since much of the learning is in terms of new language and terminology, reading comprehension is important to learning the mathematical concepts in this course. Real numbers are studied in terms of binary operations and the properties that hold in the set (i.e., commutativity, associativity). The binary operations of subtraction, division and multiplication are studied in terms of basic principles that hold in the set of reals. All basic principles learned are integrated into a chapter on algebraic manipulation. The relation of algebra to geometry is studied. Elements of logic and deductive organization are also studied.

Set theory is used as an introduction to the structure of the number system in the algebra course. The sets of integers, rational numbers and real numbers are studied in terms of the basic principles that hold and of operations within the respective sets. This leads to the introduction of the concept of the variable that the student works with throughout the course. There are a number of types of equations that the students are expected to learn how to solve. In each case, fundamental concepts and operations are introduced and the students learn to use the skills to solve equations that are already given. From this, it is expected that the students solve verbal problems involving similar concepts. There is an emphasis on integrating graphing whenever it helps students to visualize concepts or solutions to equations.

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[1] Tsunehiro, A; "A Descriptive Study of the ninth Grade Mathematics Curriculum at Stevenson Intermediate School" unpublished masters paper, University of Hawaii, Honolulu, 1966.

Algebra, geometry, and general mathematics are incorporated into a course in general mathematics, Mathematics 9X. Familiar ideas and instruments are used to introduce newer principles and terminology in these fields. Topics that students should have been exposed to are enlarged upon and newer ideas are interjected. The influence of newer trends is particularly seen when the students study the structure of the positive integers or whole numbers by looking at the historical development of various number system. Measurement is used as the basis for studying and reviewing main concepts concerning fractions, decimals, and per cents. Elements of simple descriptive statistics are introduced. Also introduced are basic concepts involved in the study of algebra such as variables, algebraic symbols, directed numbers and solutions to equations. The final part of the course deals with applications of mathematics to daily life.

Mathematics 9A and Mathematics 9U are basically remedial courses in mathematics. The distinction between the two courses is that more capable students are selected for Mathematics 9A than for Mathematics 9U. The subject matter of both courses is essentially the same but the depth of the treatment varies. The main objective of each course is to teach or review the structure of the whole and rational number systems, and to gain proficiency in operations dealing with these sets. In addition the elementary concepts of business mathematics, measurement, geometry, and algebra are introduced in Mathematics 9A.

Table A

Intercorrelations of Subtests of Orleans Algebra Prognosis Test

	Arith. Test	1	2	3	4	5	6	7	8	9	10	Total
Arith. Test	1.0000	0.5951	0.5119	0.6281	0.6265	0.6472	0.5973	0.6225	0.6322	0.6375	0.6449	0.7853
Test 1		1.0000	0.7554	0.7522	0.7386	0.6666	0.6042	0.6719	0.5292	0.5742	0.6755	0.7941
Test 2			1.0000	0.6938	0.6211	0.5446	0.5398	0.6039	0.4253	0.5091	0.6105	0.7012
Test 3				1.0000	0.7591	0.7014	0.6433	0.7247	0.6179	0.6638	0.7208	0.8483
Test 4					1.0000	0.7023	0.6312	0.6580	0.5840	0.5896	0.6851	0.8048
Test 5						1.0000	0.7005	0.7311	0.6767	0.7069	0.7535	0.8536
Test 6							1.0000	0.6762	0.6374	0.6418	0.7421	0.8096
Test 7								1.0000	0.6494	0.7132	0.7373	0.8500
Test 8									1.0000	0.7418	0.7081	0.8428
Test 9										1.0000	0.7569	0.8666
Test 10											1.0000	0.8843
Total												1.0000

Table B

Intercorrelations of Subtests of Orleans Geometry Prognosis Test

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Total
Test 1	1.0000	0.6556	0.5935	0.6533	0.4567	0.7009	0.6539	0.6246	0.7171	0.8216
Test 2		1.0000	0.6571	0.6697	0.4850	0.6648	0.5990	0.5415	0.7255	0.8201
Test 3			1.0000	0.6992	0.4184	0.5705	0.5769	0.4777	0.6961	0.7956
Test 4				1.0000	0.5062	0.6710	0.6461	0.5380	0.7573	0.8448
Test 5					1.0000	0.4980	0.4770	0.4251	0.5361	0.5940
Test 6						1.0000	0.6394	0.6597	0.7653	0.8389
Test 7							1.0000	0.5847	0.7074	0.7923
Test 8								1.0000	0.6430	0.7886
Test 9									1.0000	0.9033
Total										1.0000

Table C

Intercorrelations of 3 DAT subtests, SCAT subtests and CMT subtests

	SCAT V	SCAT Q	SCAT T	Coop Part I	Coop Part 2	Coop Part 3	Coop Part 4	DAT NA	DAT AR	DAT SR
SCAT V	1.0000	0.9609	0.9746	0.4043	0.4557	0.3357	0.3280	0.4174	0.4346	0.3839
SCAT Q		1.0000	0.9946	0.4127	0.4324	0.3076	0.3387	0.4574	0.4254	0.4135
SCAT T			1.0000	0.3771	0.4151	0.2809	0.3114	0.4226	0.4127	0.3936
Coop-Part 1				1.0000	0.7495	0.7580	0.7560	0.7473	0.5481	0.5754
Coop-Part 2					1.0000	0.7419	0.7045	0.6980	0.5571	0.5667
Coop-Part 3						1.0000	0.8127	0.6840	0.5270	0.5834
Coop-Part 4							1.0000	0.6612	0.5190	0.5600
DAT - NA								1.0000	0.6787	0.6652
DAT - AR									1.0000	0.6339
DAT - SR										1.0000

Table D

Correlations of Orleans Algebra Prognosis Test with Orleans Geometry Prognosis Test

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Total
Arith. Test	0.5594	0.5727	0.5702	0.6283	0.4199	0.5308	0.5370	0.4508	0.5753	0.6594
Test 1	0.5024	0.5183	0.6263	0.6462	0.3685	0.4321	0.5145	0.4080	0.5441	0.6295
Test 2	0.4030	0.4394	0.5304	0.5624	0.2926	0.3401	0.4159	0.3098	0.4225	0.5108
Test 3	0.5430	0.5949	0.6618	0.6736	0.3887	0.5139	0.5295	0.4614	0.6271	0.6903
Test 4	0.4672	0.5529	0.5914	0.6410	0.3787	0.4695	0.4970	0.4114	0.5503	0.6279
Test 5	0.6211	0.6623	0.6334	0.7119	0.4572	0.6077	0.5790	0.5037	0.6767	0.7460
Test 6	0.5608	0.5711	0.6019	0.6343	0.4230	0.5664	0.5631	0.5093	0.6214	0.6967
Test 7	0.6149	0.5937	0.6692	0.7200	0.4292	0.5718	0.5708	0.4819	0.6569	0.7319
Test 8	0.6483	0.6247	0.5902	0.6934	0.4232	0.6667	0.5651	0.5633	0.6811	0.7540
Test 9	0.6125	0.6255	0.6300	0.6985	0.4540	0.6304	0.5662	0.5185	0.6916	0.7459
Test 10	0.6384	0.6507	0.6721	0.7012	0.4629	0.6182	0.6009	0.5376	0.7039	0.7674
Total	0.6981	0.7146	0.7431	0.8088	0.5032	0.6811	0.6604	0.5840	0.7603	0.8465

Table E

Correlations of Orleans Algebra Prognosis Test w/SCAT subtests, CMT subtests & DAT subtests

	Arith.	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10	Total
SCAT V	0.2761	0.3812	0.3338	0.3359	0.2876	0.2868	0.2783	0.3392	0.2290	0.2507	0.2890	0.3456
SCAT Q	0.3266	0.4224	0.3794	0.3787	0.3291	0.3196	0.3115	0.3730	0.2596	0.2896	0.3303	0.3919
SCAT T	0.2926	0.4036	0.3632	0.3541	0.3049	0.2949	0.2851	0.3499	0.2309	0.2609	0.3008	0.3602
CMT-Part 1	0.5853	0.4986	0.4214	0.5801	0.5377	0.5632	0.5780	0.6087	0.6238	0.5994	0.6201	0.6964
CMT-Part 2	0.5355	0.5027	0.4155	0.5868	0.5509	0.6142	0.5814	0.5861	0.6142	0.5723	0.6462	0.6904
CMT-Part 3	0.4582	0.3650	0.2856	0.4628	0.4329	0.5121	0.5591	0.5285	0.6129	0.5670	0.5909	0.6166
CMT-Part 4	0.4830	0.3792	0.3072	0.4598	0.4337	0.5031	0.5129	0.5060	0.5691	0.5379	0.5502	0.5956
DAT-NA	0.6773	0.6026	0.5296	0.6606	0.6261	0.6488	0.6199	0.6709	0.7103	0.6882	0.7161	0.7963
DAT-AR	0.4756	0.5034	0.4770	0.5333	0.4675	0.5266	0.4766	0.5296	0.4912	0.5243	0.5438	0.6066
DAT-SR	0.4751	0.4936	0.4272	0.5003	0.4614	0.5406	0.5221	0.5554	0.5442	0.5357	0.5883	0.6265

Table F

Unrotated Factor Matrix for the 29 Variable Solutions*

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
SCAT Q	49	65	43	28	11	03	14	-02	-03	00	00	00	01	00	02	00	01	04	02	00	00	01	00	02	02	00	00	01	13
STEP M	47	66	42	30	09	04	08	-04	-05	00	00	00	-00	01	01	-00	-00	02	-01	00	01	-00	-02	-00	-01	-01	00	-03	-14
CMT Sk	78	-03	30	-30	16	-02	-01	-00	10	-03	05	07	-03	-02	-12	-03	01	-01	09	-08	-10	-14	18	-12	09	06	10	-09	-00
CMT Fa	78	-01	28	-20	16	-01	-10	-08	00	-01	-08	-13	07	10	03	06	-14	-32	08	-01	-02	04	-10	-04	-05	06	-06	07	00
CMT Appl	73	-23	42	-26	09	02	-12	-04	-06	-01	01	02	-03	00	04	-00	01	05	-10	01	05	-00	-08	-02	-10	-23	-03	-16	02
CMT Appr	71	-18	42	-27	09	-06	-12	-10	08	01	05	06	-07	06	04	-02	10	18	-07	14	-01	11	-02	10	05	10	03	15	-01
OAPT Ar	75	02	-16	-07	09	-06	32	16	34	20	06	-03	-19	-05	-06	00	06	-09	00	05	03	08	-08	01	-05	-01	02	-04	-00
OAPT Su	76	35	-32	-02	03	-01	-18	06	06	01	-02	07	03	02	-04	01	-09	05	-04	19	-04	00	14	-10	-24	-01	01	06	00
OAPT Use	66	42	-34	-07	-04	-02	-25	12	02	-10	06	24	-14	04	14	12	06	-08	00	-01	-06	-02	-07	07	09	02	-07	-08	00
OAPT Me	81	20	-28	-10	07	02	-08	-02	05	-09	-11	00	04	00	-00	-13	00	04	14	-19	04	24	04	02	04	-13	07	04	-00
OAPT Su	76	19	-31	-15	16	-04	-05	-00	06	15	-22	-08	18	-02	-04	-03	-01	19	-00	-01	07	-12	-17	-04	06	08	-05	-05	00
OAPT Su	82	-01	-22	-02	02	00	10	-06	-13	12	-08	-15	09	20	-01	-01	11	-07	-01	18	-13	00	13	09	12	-09	-04	-05	-00
OAPT Re	78	-01	-15	-08	12	03	-01	03	-35	20	13	-14	-18	-12	-00	-18	03	08	01	-10	01	06	04	-08	-00	03	-04	04	-00
OAPT Po	82	08	-18	-04	00	03	05	-06	-12	-10	17	03	05	10	-24	-16	13	-05	-15	-15	-03	-04	-07	08	-12	06	-01	01	00
OAPT Pr	80	-21	-05	-08	02	10	27	07	-03	-07	-04	08	10	-02	09	11	-25	10	-06	-06	-18	06	-00	11	-03	05	03	-05	-00
OAPT Add	81	-11	-14	-06	-03	03	26	00	-11	-18	10	10	04	-03	17	-17	06	01	08	11	-01	01	-06	-22	02	01	-07	03	-00
OAPT Sum	86	-01	-17	-08	01	02	02	00	-19	01	04	01	-06	-00	10	-06	-10	-07	08	08	20	-15	-01	14	-00	-00	20	01	00
OGPT Ax	76	-23	00	25	-10	15	-01	08	07	-02	09	-00	-13	36	-11	02	-18	10	02	-01	10	-01	-01	-07	07	-01	-04	01	00
OGPT Rdg	77	-15	-06	21	-10	01	03	-26	07	13	-25	05	-19	06	21	-05	06	-02	-11	-14	-04	-07	04	-05	-06	02	02	03	00
OGPT Ki	78	03	-15	11	-09	05	-06	-34	10	-16	06	-14	-16	-24	-12	-01	-12	02	02	08	-08	-04	-04	04	06	-05	-05	01	00
OGPT Co	84	-02	-10	09	-06	-02	03	-10	07	-10	04	01	19	-03	-02	27	08	-06	-19	00	08	02	-02	-11	08	-04	14	03	-00
OGPT Un	56	-17	01	29	07	-72	00	06	-08	-07	-01	00	-02	01	-03	-02	-06	01	00	-01	-02	00	-00	00	00	-01	-00	-00	00
OGPT Bi	76	-32	11	22	-04	08	00	03	-01	01	-10	15	06	-02	-11	16	16	04	27	00	-07	-09	-07	05	-09	-05	-00	07	-00
OGPT Ge	73	-15	01	29	-00	03	-20	07	14	22	33	-08	20	-05	17	-10	-02	-00	01	-06	-08	-04	-00	01	00	-03	00	01	00
OGPT Ge	65	-27	08	32	20	20	-14	36	-04	-14	-19	-09	-07	-13	-03	-10	04	-06	-13	04	-02	01	-00	-00	04	02	02	01	-00
OGPT Su	83	-24	01	21	-02	03	-05	-14	03	-04	00	02	08	-05	00	01	05	-01	06	04	18	12	13	05	-05	16	07	-18	-00
DAT N	85	05	14	-19	-07	-04	12	14	09	-04	-02	02	05	-10	02	02	-02	-01	-06	-07	15	-11	16	07	03	-06	-19	13	-00
DAT Abs	69	18	18	-18	-47	-07	-03	17	02	-14	-03	-32	-02	04	06	02	08	07	05	-02	-03	-01	-01	-00	-06	05	05	-03	-00
DAT Sp	72	05	23	-05	-44	-00	-04	04	-13	26	-08	20	03	-11	-14	-10	-09	-08	-03	03	-01	08	-03	-03	07	00	00	-00	00

*Decimals are omitted

Table G

Mathematics Test Variables and Factor Loadings on the Unrotated Factor Matrix*

	Factors			
Variables	I	II	III	IV
SCAT Quantitative	495	785	136	-208
STEP Mathematics	491	793	126	-218
CMT Skills	773	054	283	275
CMT Facts, Terms & Concepts	776	061	252	177
CMT Applications	730	-075	468	219
CMT Appreciation	709	-033	432	225
OAPT Arithmetic Test	744	-036	-130	057
OAPT Substitution in Monomials	747	174	-408	068
OAPT Use of Exponents	642	207	-423	117
OAPT Meaning of Exponents	802	057	-316	112
OAPT Substitution in Monomials with Exponents	749	040	-326	152
OAPT Substitution in Binomials with Exponents	822	-092	-188	011
OAPT Representation of Relations	775	-070	-113	060
OAPT Positive and Negative Numbers	820	-001	-182	041
OAPT Problems	800	-199	036	027
OAPT Addition of Like Terms	810	-148	-074	025
OAPT Summary Test	857	-083	-143	066
OGPT Axioms	757	-176	076	-242
OGPT Reading Angles	765	-136	-002	-197
OGPT Kinds of Angles	769	-020	-141	-101
OGPT Complementary and Supplementary Angles	843	-061	-085	-099
OGPT Understanding Geometrical Relationships	554	-101	053	-163
OGPT Bisection	766	-216	206	-242
OGPT Geometrical Notation	729	-102	051	-236
OGPT Geometrical Problems	651	-159	139	-259
OGPT Summary Test	837	-192	095	-237
DAT Numerical Ability	848	072	109	186
DAT Abstract Reasoning	672	172	075	151
DAT Space Relations	709	094	157	042
Eigenvalues	16.433	1.819	1.679	1.111

*Decimals are omitted

Table H

Unrotated Factor Matrix for the Eight Variable Solution*

Variables	1	2	3	4	5	6	7	8
1	-66	-72	-06	-01	-00	-01	-00	-14
2	-66	-73	-05	01	-00	02	-00	14
3	-82	20	-12	-16	46	10	-06	-00
4	-86	27	-23	01	-25	-04	-25	00
5	-84	27	-28	10	-14	22	20	-00
6	-88	18	-02	-12	00	-38	13	01
7	-78	08	49	-29	-15	14	00	-00
8	-79	12	33	47	10	-02	-02	-00

*Decimals are omitted

Table I

Mathematics Test Variables and Factor Loadings on the Rotated Factor Matrix*

Variables	Factors I	II	III	IV	V	h^2
SCAT Quantitative	136	922	173	-199	-117	952
STEP Mathematics	129	926	149	-193	-128	950
CMT Skills	262	187	761	-360	-136	833
CMT Facts, Terms & Concepts	325	226	690	-354	-123	774
CMT Applications	366	088	810	-162	-186	860
CMT Appreciation	323	117	793	-173	-185	811
OAPT Arithmetic Test	365	076	314	-601	-093	608
OAPT Substitution in Monomials	237	252	099	-813	-144	811
OAPT Use of Exponents	109	238	037	-797	-200	746
OAPT Meaning of Exponents	292	143	237	-786	-124	796
OAPT Substitution in Monomials with Exponents	235	108	254	-796	-027	766
OAPT Substitution in Binomials with Exponents	455	038	269	-654	-155	734
OAPT Representation of Relations	399	057	358	-606	-063	662
OAPT Positive and Negative Numbers	397	121	280	-666	-187	730
OAPT Problems	510	-052	438	-464	-161	697
OAPT Addition of Like Terms	477	-021	341	-554	-211	697
OAPT Summary Test	441	046	349	-654	-183	780
OGPT Axioms	714	064	224	-298	-230	707
OGPT Reading Angles	653	076	194	-386	-241	678
OGPT Kinds of Angles	499	141	158	-549	-247	657
OGPT Complementary and Supplementary Angles	559	126	250	-542	-245	745
OGPT Understanding Geometrical Relationships	601	118	156	-205	-020	443
OGPT Bisection	744	054	343	-207	-192	754
OGPT Geometrical Notation	686	156	201	-310	-142	652
OGPT Geometrical Problems	721	133	274	-190	078	655
OGPT Summary Test	732	077	309	-354	-186	799
DAT Numerical Ability	333	189	546	-484	-344	799
DAT Abstract Reasoning	180	220	338	-359	-700	815
DAT Space Relations	349	204	351	-270	-655	790
Eigenvalues	16.432	1.824	1.675	1.111	.672	
Cumulative Proportion of Total Variance	.566	.629	.687	.725	.748	

*Decimals are omitted

Table 3

Mathematics Test Variables and Factor Loadings on the Rotated Factor Matrix*

Variables	I	II	III	IV	V	VI	VII	h^2
SCAT Quantitative	115	935	164	-191	-115	-048	079	973
STEP Mathematics	121	930	149	-200	-127	-035	026	961
CMT Skills	232	188	751	-314	-138	-074	235	833
CMT Facts, Terms & Concepts	307	212	697	-335	-126	-090	147	785
CMT Applications	363	078	814	-141	-186	-056	130	875
CMT Appreciation	295	097	803	-159	-193	-126	106	832
OAPT Arithmetic Test	261	135	245	-461	-087	-165	568	718
OAPT Substitution in Monomials	240	212	130	-830	-148	-081	083	845
OAPT Use of Exponents	132	183	083	-842	-208	-045	-012	812
OAPT Meaning of Exponents	287	128	243	-761	-121	-055	217	804
OAPT Substitution in Monomials with Exponents	203	094	258	-764	-031	-108	241	772
OAPT Substitution in Binomials with Exponents	405	062	237	-569	-147	-116	401	745
OAPT Representation of Relations	377	060	348	-556	-057	-077	277	664
OAPT Positive and Negative Numbers	368	136	256	-598	-179	-071	344	735
OAPT Problems	460	014	365	-322	-139	-036	559	784
OAPT Addition of Like Terms	410	036	274	-418	-197	-094	548	769
OAPT Summary Test	411	057	329	-587	-175	-088	340	781
OGPT Axioms	719	079	210	-256	-211	-064	219	731
OGPT Reading Angles	609	088	177	-332	-233	-177	267	679
OGPT Kinds of Angles	494	136	158	-523	-239	-086	192	664
OGPT Complementary and Supplementary Angles	506	134	234	-485	-242	-183	299	747
OGPT Understanding Geometrical Relationships	327	073	182	-188	-079	-875	124	969
OGPT Bisection	720	071	326	-156	-178	-135	239	762
OGPT Geometrical Notation	681	126	230	-330	-139	-177	021	694
OGPT Geometrical Problems	755	126	286	-191	097	-037	062	720
OGPT Summary Test	702	078	306	-318	-178	-179	210	803
DAT Numerical Ability	276	214	510	-399	-343	-110	379	816
DAT Abstract Reasoning	165	211	335	-338	-705	-066	144	821
DAT Space Relations	344	199	349	-251	-653	-066	132	792
Eigenvalues	16.432	1.824	1.675	1.111	.672	.646	.538	
Cumulative Proportion of Total Variance	.566	.629	.687	.725	.748	.771	.789	

*Decimals are omitted

Table K

Summary Statistics for Mathematics Final Grade and Teacher Assigned Stanine Scores

Criterion	Boys			Girls			Total		
	n	Mean	S.D.	n	Mean	S.D.	n	Mean	S.D.
Mathematics Final Grade	216	2.1	.95	246	2.0	1.00	462	2.0	.98
Teacher Assigned Stanine Scores	224	4.9	1.90	243	5.1	1.91	467	5.0	1.91